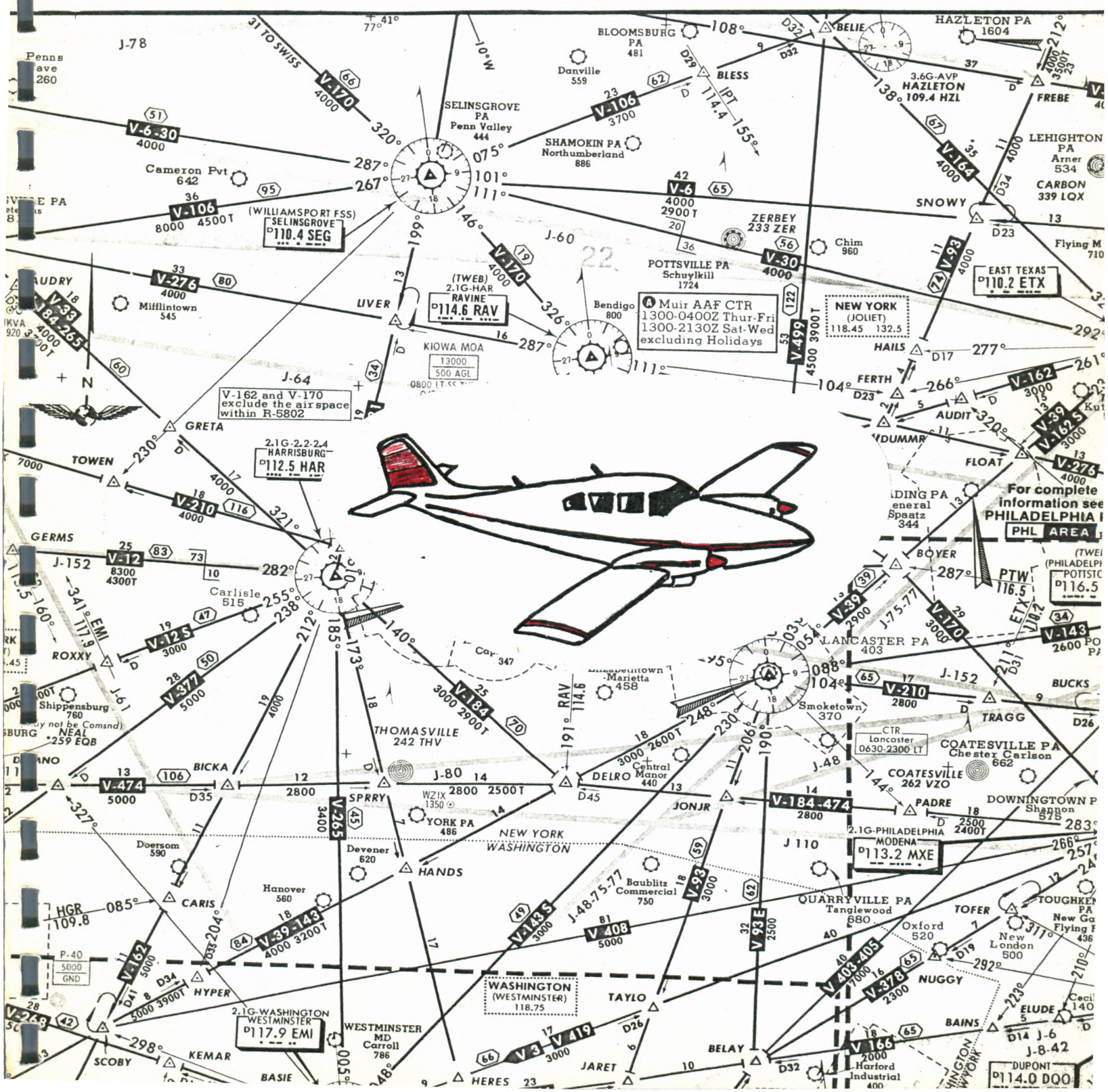
AREA NAVIGATION

RNAV Program for the HP 41C/CV

by

Robert A. Tims

and Other Programs



AIR NAVIGATION/*RNAV*
NAVIGATION FORMULAS

DISTANCE: The Great Circle distance between fixes is calculated by the following formulas:

$$d=COS^{-1}[SIN(LAT)SIN(LAT')+COS(LAT)COS(LAT')COS(LNG-LNG')]$$

$$(Distance\ in\ NM)=60d$$

Where: LAT' and LNG' are the coordinates of the "from" fix.
LAT and LNG are the coordinates of the "to" fix.

Using Atari functions the formula for d is:

$$d=90-TAN^{-1}[X/(1-X^2)^2]$$
$$X=[SIN(LAT)SIN(LAT')+COS(LAT)COS(LAT')COS(LNG-LNG')]$$

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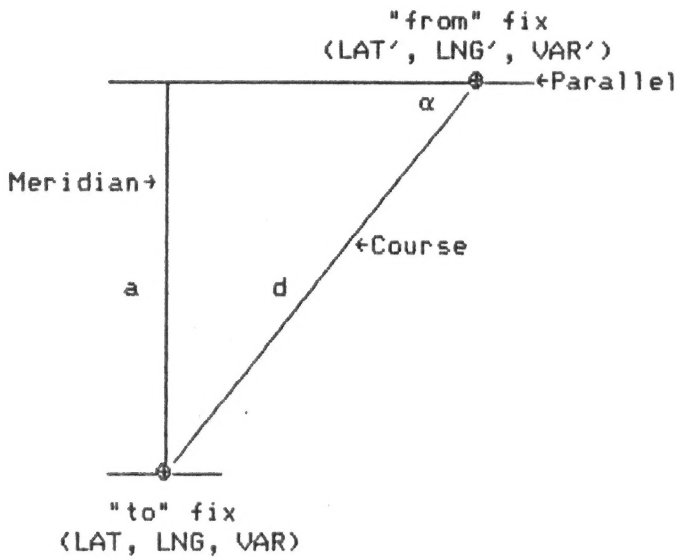
COURSE: The course is derived by finding the angle "α" between the parallel passing through the "from" fix and the Course. The angle "α" is then corrected for East/West direction of travel.

A diagram illustrating the relationships is given below with the formulas:



$a=COS^{-1}(Y)$ Atari functions: $a=90-TAN^{-1}[Y/(1-Y^2)^2]$
Where $Y=[SIN(LAT)SIN(LAT')+COS(LAT)COS(LAT')]$

$d=COS^{-1}(X)$ Atari functions: $d=90-TAN^{-1}[X/(1-X^2)^2]$
Where $X=[SIN(LAT)SIN(LAT')+COS(LAT)COS(LAT')COS(LNG-LNG')]$

$\alpha=SIN^{-1}(A)$ Atari functions: $\alpha=TAN^{-1}[A/(1-A^2)^2][SGN(LAT-LAT')]$
Where $A=SIN(a)/SIN(d)$

If $(LNG'-LNG)<0$ then $TC=270+\alpha$ (Westerly direction of travel.)
If $(LNG'-LNG)\geq 0$ then $TC=90-\alpha$ (Easterly direction of travel.)

MC=TC+VAR', and if VAR≠VAR' then MC=TC+VAR'/TC+VAR

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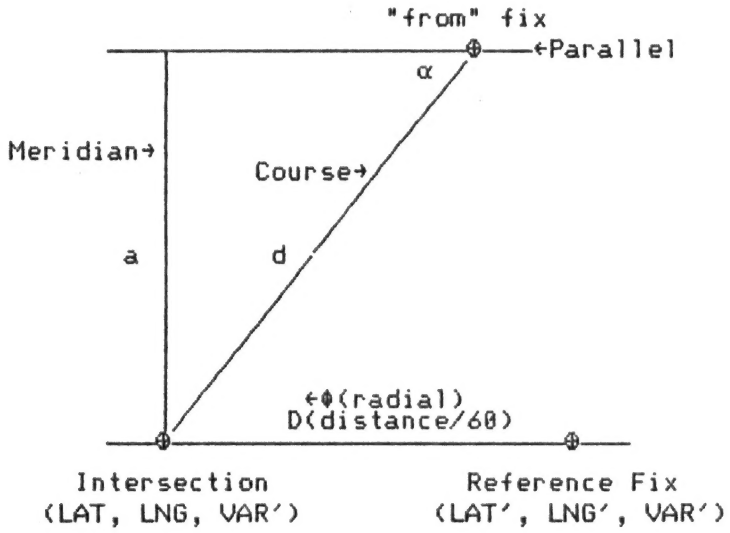
INTERSECTIONS: Intersections are always defined by the radial (magnetic bearing) and distance from a known fix (reference navaid). The program uses this information along with the coordinates of the reference fix to derive the coordinates of the intersection. These coordinates are then used the same as the coordinates of any other fix.

A diagram illustrating the relationships and the formulas is given below:



$LAT=LAT'+SIN^{-1}[(SIN\ D)(COS\ \Phi)]$

$LNG=LNG'-SIN^{-1}[[(SIN\ D)(SIN\ \Phi)]/[COS[(LAT+LAT')/2]]$

The formulas using Atari functions:

$LAT=LAT'+TAN^{-1}[X/(1-X^2)^2]$
Where $X=(SIN\ D)(COS\ \Phi)$

$LNG=LNG'-TAN^{-1}[Y/(1-Y^2)^2]$
Where $Y=[(SIN\ D)(SIN\ \Phi)]/COS[(LAT+LAT')/2]$

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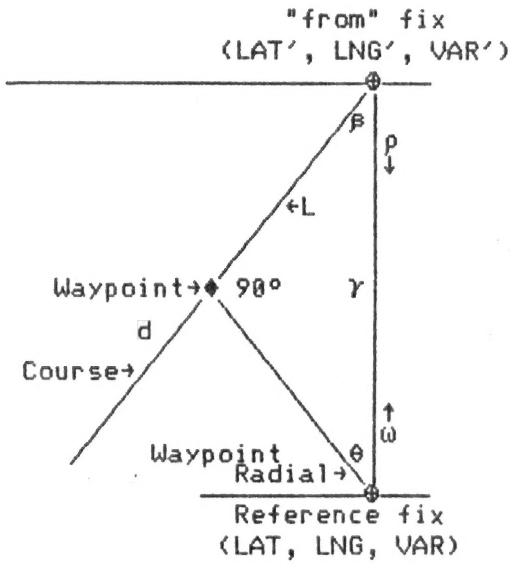
WAYPOINTS: Waypoints are calculated by deriving the bearing and distance of the waypoint from the reference navaid along the radial that is perpendicular to the course. This will form a spherical right triangle between the waypoint, reference navaid, and the "from" fix:



The bearing (ρ) and distance angle (γ) from the "from" fix to the reference fix is calculated first. The angle, γ, is calculated with the same formula as used to find the course distance angle (d), and ρ is calculated the same way as the true course. The bearing (ω) from the reference fix to the "from" fix and is the reciprocal of ρ, ω=ρ±180. The angle, β, is the difference between the bearing (ρ) and the true course. This angle is expressed as:

$$-180° \leq \beta \leq 180°$$

If $\beta > |90°|$ then the way point will not be usable because it will occur the other side of the "from" fix.

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The distance (L) between the "from" fix and the waypoint is calculated with the following formula:

$L=60[SIN^{-1}[SIN(|\theta|)SIN(\gamma)]]$ $\theta=90-\beta$

Using the Atari functions the formula becomes:

$L=60[TAN^{-1}[X/(1-X^2)^2]]$
Where $X=SIN(|\theta|)SIN(\gamma)$

The waypoint radial and distance are calculated with the following formulas:

Waypoint radial = $\omega+SGN(\beta)(\theta)+VAR$

Waypoint distance = $60[SIN^{-1}[SIN(|\beta|)SIN(\gamma)]$

Using Atari functions:

Waypoint distance = $60[TAN^{-1}[Y/(1-Y^2)^2]$
Where $Y=SIN(|\beta|)SIN(\gamma)]$

TERMINAL WAYPOINTS

The terminal waypoints are calculated by letting the reference navaid be the "from" fix and the terminal airport be the "to" fix. The same formulas used to find the true course and distance are used. The reference navaid's magnetic variation is added to the bearing from the reference fix to the terminal airport to obtain the proper radial. The reference navaid's magnetic variation is also used to obtain the magnetic course from the departure airport or to the destination airport as the case may be.

CURRENT AIRWAY PROGRAM

HP41C AIRNAV Program

by Robert A. Tims

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```
01♦LBL "AIRNAV"
02 19
03 PSIZE
04♦LBL 01
05 32
06 X<>F
07 SF 21
08 SF 23
09 CF 29
10 0
11 STO 17
12 "FROM?"
13 XEQ 04
14 CF 05
15 RCL 05
16 STO 18
17 FC?C 08
18 XEQ 09
19♦LBL 20
20 CLA
21 ARCL 05
22 ASTO 01
23 "⊢ TO?"
24 CF 23
25 XEQ 04
26 FC? 23
27 GTO 22
28 ADV
29 XEQ 08
30 STO 00
31 RCL 10
32 FIX 0
33 RND
34 ST+ 17
35 STO 15
36 RCL 05
37 STO 11
38 RCL 06
39 STO 12
40 RCL 07
41 STO 13
42 RCL 08
43 STO 14
44 0
45 STO 16
46 SF 06
47♦LBL 21
48 CLA
49 ARCL 01
50 "⊢ TO WP?"
51 CF 23
52 XEQ 04
53 FC? 23
54 GTO 02
55 "WP:"
56 ARCL 05
57 ASTO 05
58 XEQ 08
59 STO 10
60 180
61 +
62 90
63 RCL 00
64 RCL 10
65 -
66 1
67 P-R
68 R-P
69 RDN
70 X<0?
71 SF 05
72 ABS
73 STO 10
74 -
75 X<=0?
76 GTO 21
77 STO 07
78 FC?C 05
79 CHS
80 +
81 RCL 08
82 +
83 X<> 07
84 SIN
85 RCL 09
86 *
87 ASIN
88 60
89 *
90 FIX 0
91 RND
92 RCL 15
93 X<=Y?
94 GTO 21
95 X<>Y
96 RCL 16
97 X>Y?
98 GTO 21
99 RDN
100 ENTER↑
101 X<> 16
102 XEQ 03
103 ADV
104 FIX 1
105 CLA
106 ARCL 05
107 XEQ 15
108 CLA
109 RCL 07
110 XEQ 10
111 "⊢, "
112 RCL 10
113 SIN
114 RCL 09
115 *
116 ASIN
117 60
118 *
119 ARCL X
120 "⊢ NM"
121 XEQ 15
122 RCL 00
123 STO 04
124 ADV
125 RCL 05
126 STO 01
127 GTO 21
128♦LBL 02
129 CF 06
130 RCL 11
131 STO 05
132 RCL 14
133 STO 08
134 RCL 15
135 RCL 16
136 XEQ 03
137 RCL 12
138 STO 02
139 RCL 13
140 STO 03
141 RCL 08
142 STO 04
143 GTO 20
```

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```
144♦LBL 03
145 FIX 0
146 -
147 CLA
148 ARCL 01
149 FC? 55
150 GTO 00
151 ACA
152 CLA
153 SF 13
154 " TO "
155 ACA
156 CF 13
157 CLA
158♦LBL 00
159 FC? 55
160 "⊦-"
161 ARCL 05
162 XEQ 15
163 "MC="
164 RCL 00
165 RCL 04
166 +
167 STO 06
168 XEQ 10
169 RDN
170 CLX
171 RCL 06
172 RCL 00
173 RCL 08
174 +
175 X≠Y?
176 "⊦/"
177 X≠Y?
178 XEQ 10
179 XEQ 15
180 "DIST="
181 ARCL T
182 "⊦ NM"
183 GTO 15
184♦LBL 04
185 8
186 FS? 05
187 4
188♦LBL 05
189 STO 09
190♦LBL 06
191 AON
192 TONE 8
193 PROMPT
194 AOFF
195 FC? 23
196 RTN
197 FC? 06
198 CF 08
199 ASTO 05
200 4
201 ALENG
202 X>Y?
203 GTO 00
204 SF 25
205 XEQ IND 05
206 FC?C 25
207 GTO 01
208♦LBL 07
209 STO IND 09
210 DSE 09
211 RDN
212 HR
213 STO IND 09
214 DSE 09
215 RDN
216 HR
217 STO IND 09
218 RTN
219♦LBL 01
220 CF 22
221 "⊦ LAT?"
222 TONE 8
223 PROMPT
224 CLA
225 ARCL 05
226 FC?C 22
227 GTO 00
228 "⊦ LNG?"
229 TONE 8
230 PROMPT
231 CLA
232 ARCL 05
233 FC?C 22
234 GTO 00
235 "⊦ VAR?"
236 TONE 8
237 PROMPT
238 FS? 22
239 GTO 07
240♦LBL 00
241 FC? 06
242 FS? 07
243 GTO 06
244 RCL 02
245 STO 06
246 RCL 03
247 STO 07
248 RCL 04
249 STO 08
250 RCL 05
251 STO 11
252 SF 07
253 CF 23
254 8
255 "NEED FIX?"
256 XEQ 05
257 CF 07
258 RCL 01
259 FS? 23
260 RCL 05
261 STO 10
262 RCL 11
263 STO 05
264 CF 22
265 "RADIAL?"
266 TONE 8
267 PROMPT
268 CLA
269 ARCL 05
270 SF 23
271 FC?C 22
272 GTO 04
273 "DME?"
274 TONE 8
275 PROMPT
276 CLA
277 ARCL 05
278 FC? 22
279 GTO 04
280 X<>Y
281 FC? 55
282 GTO 00
283 ADV
284 "⊦ * WP:"
285 ARCL 10
286 PRA
287 CLA
288 FIX 1
289 XEQ 10
290 RDN
```

HP41C AIRNAV Program

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```
291♦LBL 00
292 ", "
293 ARCL Y
294 "├ NM"
295 FS? 55
296 XEQ 15
297 RCL 08
298 FS? 05
299 STO 04
300 -
301 STO 09
302 RDN
303 60
304 /
305 SIN
306 STO 10
307 RCL 09
308 SIN
309 *
310 ASIN
311 RCL 09
312 COS
313 RCL 10
314 *
315 ASIN
316 RCL 06
317 +
318 STO 06
319 FS? 05
320 STO 02
321 LASTX
322 +
323 2
324 /
325 COS
326 /
327 ST- 07
328 RCL 07
329 FS? 05
330 STO 03
331 SF 08
332 RTN
333♦LBL 08
334 RCL 06
335 SIN
336 RCL 02
337 SIN
338 *
339 STO 09
340 RCL 06
341 COS
342 RCL 02
343 COS
344 *
345 STO 10
346 +
347 ACOS
348 SIN
349 RCL 06
350 RCL 02
351 -
352 SIGN
353 *
354 RCL 10
355 RCL 07
356 RCL 03
357 -
358 X<0?
359 SF 05
360 COS
361 *
362 RCL 09
363 +
364 ACOS
365 SIN
366 STO 09
367 LASTX
368 60
369 *
370 STO 16
371 RDN
372 /
373 ASIN
374 FS? 05
375 CHS
376 90
377 +
378 180
379 FS?C 05
380 CLX
381 +
382 RTN
383♦LBL 09
384 RCL 05
385 STO 01
386 CLA
387 ARCL 05
388 "├ WP?"
389 SF 06
390 CF 23
391 XEQ 04
392 CF 06
393 FC? 23
394 RTN
395 ADV
396 "WP:"
397 ARCL 05
398 ASTO 05
399 CLA
400 ARCL 01
401 "├ * "
402 ARCL 05
403 XEQ 15
404 CLA
405 RCL 01
406 STO 05
407 XEQ 09
408 180
409 +
410 RCL 08
411 +
412 FIX 1
413 XEQ 10
414 "├, "
415 ARCL 10
416 "├ NM"
417 GTO 15
418♦LBL 10
419 RND
420 360
421 MOD
422 X≠0?
423 GTO 00
424 LASTX
425 +
426♦LBL 00
427 100
428 X>Y?
429 "├0"
430 SQRT
431 X>Y?
432 "├0"
433 ARCL Y
434 FC? 55
435 RTN
436 ACA
437 "a"D"
438 ASTO X
439 ACSPEC
440 CLA
441 RTN
442♦LBL 22
443 FC?C 08
444 XEQ 09
445 ADV
446 CLA
447 ARCL 18
448 "├ TO "
449 ARCL 11
450 AVIEW
451 FIX 0
452 "T/DIST "
453 ARCL 17
454 FS? 55
455 "├ NM"
456♦LBL 15
457 FC? 55
458 GTO 00
459 ACA
460 PRBUF
461 RTN
462 GTO 01
463♦LBL 09
464 TONE 8
465 AVIEW
466 END
```

RVAV

(USES Extended Memory)

```
LBL'DAL
LBL'ADS
LBL'BUJ
LBL'PRX
LBL'SLR
LBL'UIM
END          155 BYTES
```

```
01♦LBL "DAL"
02 32.5051
03 96.5142
04 -8
05 RTN
06♦LBL "ADS"
07 32.5806
08 96.501
09 -8
10 RTN
11♦LBL "BUJ"
12 33.1659
13 96.2153
14 -8
15 RTN
16♦LBL "PRX"
17 33.3232
18 95.2653
19 -8
20 RTN
21♦LBL "SLR"
22 33.1155
23 95.3233
24 -8
25 RTN
26♦LBL "UIM"
27 32.5249
28 95.22
29 -8
30 END
```

```
LBL'OKC
LBL'ADM
LBL'TUL
LBL'KOKC
END          107 BYTES
```

```
01♦LBL "OKC"
02 35.2633
03 97.4621
04 -9
05 RTN
06♦LBL "ADM"
07 34.1241
08 97.1005
09 -9
10 RTN
11♦LBL "TUL"
12 36.1146
13 95.4716
14 -8
15 RTN
16♦LBL "KOKC"
17 35.2335
18 97.3602
19 -9
20 END
```

```
LBL'CPS
LBL'FAM
LBL'STL
LBL'TOY
END          112 BYTES
```

```
01♦LBL "CPS"
02 38.3418
03 90.0934
04 -3
05 RTN
06♦LBL "FAM"
07 37.4024
08 90.1402
09 -5
10 RTN
11♦LBL "STL"
12 38.5138
13 90.2856
14 -5
15 RTN
16♦LBL "TOY"
17 38.4421
18 89.5507
19 -4
20 END
```

```
LBL'BHM
LBL'VUZ
LBL'HAB
END          79 BYTES
```

```
01♦LBL "BHM"
02 33.335
03 86.4516
04 -2
05 RTN
06♦LBL "VUZ"
07 33.4012
08 86.5359
09 -2
10 RTN
11♦LBL "HAB"
12 34.1142
13 88.0045
14 -2
15 END
```

```
LBL'BNA
LBL'GHM
END          52 BYTES
```

```
01♦LBL "BNA"
02 36.071
03 86.4057
04 -2
05 RTN
06♦LBL "GHM"
07 35.5002
08 87.2706
09 3
10 END
```

```
LBL'AEX
LBL'ESF
LBL'MLU
LBL'SHV
END          103 BYTES
```

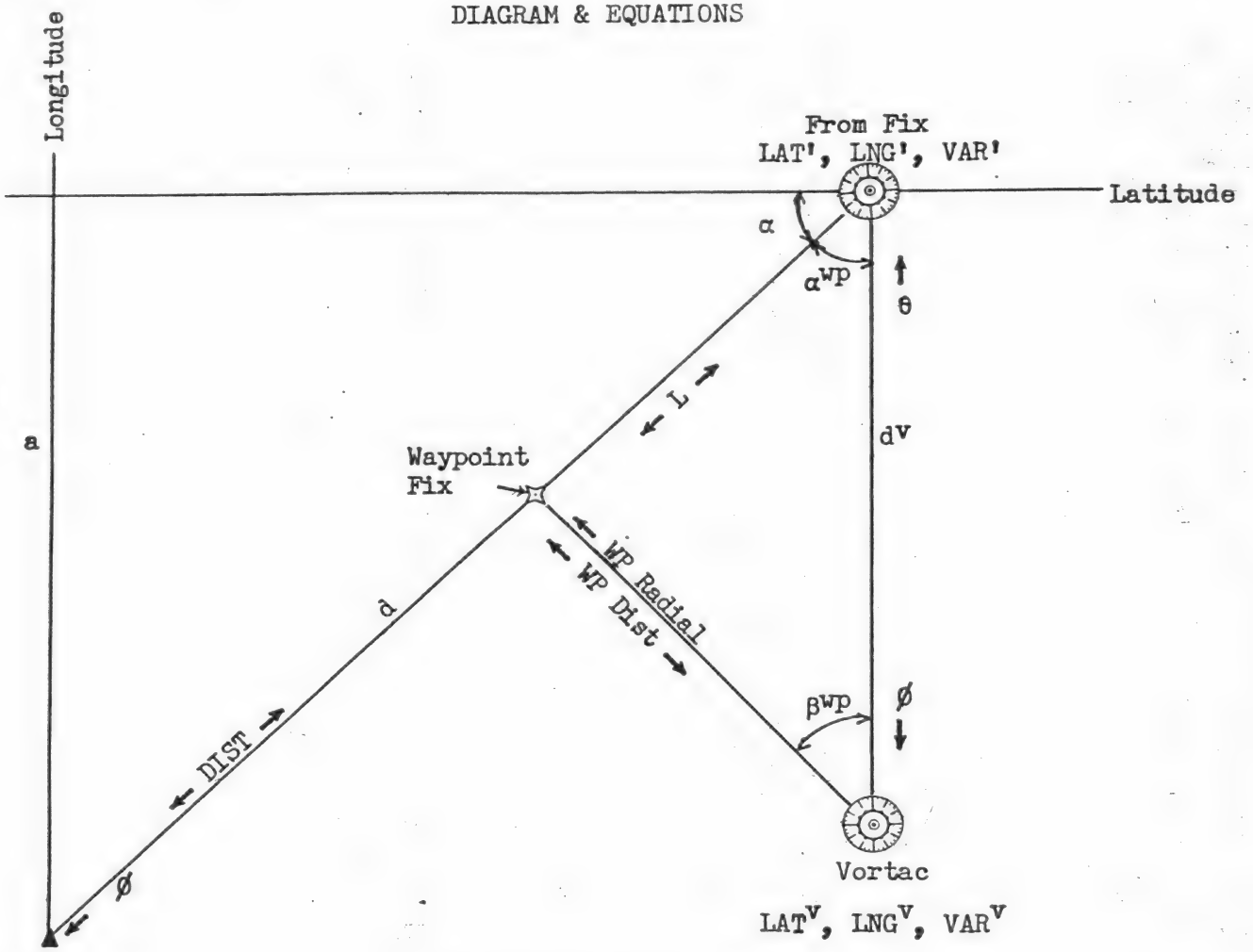
```
01♦LBL "AEX"
02 31.1523
03 92.3002
04 -7
05 RTN
06♦LBL "ESF"
07 31.234
08 92.1744
09 -7
10 RTN
11♦LBL "MLU"
12 32.31
13 92.0209
14 -6
15 RTN
16♦LBL "SHV"
17 32.4616
18 93.4835
19 -7
20 END
```

```
LBL'GLH
LBL'GRW
LBL'JAN
LBL'KJAN
END          106 BYTES
```

```
01♦LBL "GLH"
02 33.3124
03 90.5858
04 -4
05 RTN
06♦LBL "GRW"
07 33.2749
08 90.1638
09 -3
10 RTN
11♦LBL "JAN"
12 32.3026
13 90.1003
14 -5
15 RTN
16♦LBL "KJAN"
17 32.184
18 90.0433
19 -5
20 END
```

```
1. JBR
LAT: 35.4952
LNG: 90.3847
VAR: -4
2. AGO
LAT: 33.1400
LNG: 93.1300
VAR: -6
3. ARG
LAT: 36.0636
LNG: 90.5713
VAR: -4
4. BVX
LAT: 35.4328
LNG: 91.3904
VAR: -4
5. CNG
LAT: 37.0031
LNG: 88.5013
VAR: -3
6. CGI
LAT: 37.1339
LNG: 89.3420
VAR: -3
7. DGD
LAT: 37.0124
LNG: 92.5236
VAR: -6
8. ELD
LAT: 33.1522
LNG: 92.4438
VAR: -7
9. FSM
LAT: 35.2318
LNG: 94.1617
VAR: -6
10. FSM:
LAT: 35.2010
LNG: 94.2203
VAR: -6
11. FYV
LAT: 36.0018
LNG: 94.1012
VAR: -7
12. F43
LAT: 33.1100
LNG: 92.4000
VAR: -5
13. GQE
LAT: 35.2049
LNG: 90.2841
VAR: -4
14. HOT
LAT: 34.2843
LNG: 93.0526
VAR: -6
15. HRO
LAT: 36.1906
LNG: 93.1247
VAR: -6
16. HRO:
LAT: 36.1541
LNG: 93.0916
VAR: -6
17. H37
LAT: 36.1025
LNG: 94.0715
VAR: -7
18. JBR:
LAT: 35.5229
LNG: 90.3518
VAR: -4
19. JKS
LAT: 35.3556
LNG: 88.2132
VAR: -2
20. LIT
LAT: 34.4039
LNG: 92.1049
VAR: -5
21. LIT:
LAT: 34.4349
LNG: 92.1359
VAR: -5
22. MAW
LAT: 36.3318
LNG: 89.5441
VAR: -3
23. MEM
LAT: 35.0345
LNG: 89.5853
VAR: -3
24. MKL
LAT: 35.3613
LNG: 88.5437
VAR: -2
25. MKO
LAT: 35.3930
LNG: 95.2145
VAR: -7
26. MON
LAT: 33.3343
LNG: 91.4256
VAR: -4
27. M03
LAT: 35.0442
LNG: 92.2535
VAR: -6
28. M07
LAT: 35.1245
LNG: 91.4412
VAR: -5
29. PAH
LAT: 37.0339
LNG: 88.4625
VAR: -3
30. PBF
LAT: 34.1448
LNG: 91.5534
VAR: -4
31. PBF:
LAT: 34.1028
LNG: 91.5608
VAR: -4
32. PGO
LAT: 34.4049
LNG: 94.3632
VAR: -7
33. ROG
LAT: 36.2210
LNG: 94.0630
VAR: -7
34. RSN
LAT: 32.3045
LNG: 92.3745
VAR: -6
35. RZC
LAT: 36.1447
LNG: 94.0716
VAR: -7
36. SGF
LAT: 37.2121
LNG: 93.2002
VAR: -7
37. TXK
LAT: 33.3050
LNG: 94.0423
VAR: -7
38. TXK:
LAT: 33.2713
LNG: 93.5927
VAR: -7
39. 2M9
LAT: 36.2200
LNG: 92.2800
VAR: -6
40. 3R8
LAT: 31.4415
LNG: 93.0545
VAR: -7

DATA REG=160
CHAR=908
EM REG=130
                    EMDIR
RNAV    P124
F       A130
```

DIAGRAM & EQUATIONS



WP Radial = $\theta^V \pm \beta^{WP} + VAR^V$

WP Dist = $60 \sin^{-1} (\sin |\alpha^{WP}| \sin d^V)$

$$LAT = LAT' + \sin^{-1}[(\sin d)(\cos \phi)]$$

$$LNG = LNG' - \sin^{-1} \frac{(\sin d)(\sin \phi)}{\cos [(LAT + LAT')/2]}$$

$$DIST = 60d$$

$$d = \cos^{-1}[\sin LAT \sin LAT' + \cos LAT \cos LAT' \cos (LNG - LNG')]$$

$$\alpha = \sin^{-1}\left(\frac{\sin a}{\sin d}\right)$$

$$\phi = TC = 270^\circ + \alpha \text{ or } 90^\circ - \alpha$$

$$a = \cos^{-1}(\sin LAT \sin LAT' + \cos LAT \cos LAT')$$

```
LBL'RNAV
END          867 BYTES
```

```
01♦LBL "RNAV"
02 19
03 PSIZE
04♦LBL 01
05 32
06 X<>F
07 SF 21
08 SF 23
09 CF 29
10 0
11 STO 17
12 "F"
13 SEEKPTA
14 "FROM?"        Departure point
15 XEQ 04         Coordinates
16 CF 05
17 RCL 05
18 STO 18         Departure ident.
19 FC?C 08        If fix isn't a WP
20 XEQ 09         Prompt for a terminal WP
21♦LBL 20         PRIMARY LEGS
22 CLA
23 ARCL 05
24 ASTO 01
25 "⊢ TO?"
26 CF 23
27 XEQ 04         Coordinates
28 FC? 23         Input declined?
29 GTO 22
30 ADV
31 XEQ 08         True course & distance
32 STO 00         Ø
33 RCL 10         Distance
34 FIX 0
35 RND
36 ST+ 17         Total distance sum
37 STO 15         Leg distance
38 RCL 05
39 STO 11
40 RCL 06         Save ending coordinates
41 STO 12
42 RCL 07
43 STO 13
44 RCL 08
45 STO 14
46 0
47 STO 16
48 SF 06          Computing waypoints
49♦LBL 21         SECONDARY LEGS
50 CLA
51 ARCL 01
52 "⊢ TO WP?"
53 CF 23
54 XEQ 04         Coordinates
55 FC? 23         Waypoint declined?
56 GTO 02
57 "WP:"
58 ARCL 05
59 ASTO 05
60 XEQ 08         Ø^V
61 STO 10
62 180
63 +              θ^V
64 90
65 RCL 00         Ø
66 RCL 10
67 -              α^WP
68 1
69 P-R
70 R-P            -180 ≤ α^WP ≤ 180
71 RDN
72 X<0?
73 SF 05
74 ABS            |α^WP|
75 STO 10
76 -              β^WP
77 X<=0?          WP behind starting fix?
78 GTO 21
79 STO 07
80 FC?C 05
81 CHS
82 +
83 RCL 08         Var^V
84 +              Waypoint radial
85 X<> 07         β^WP
86 SIN
87 RCL 09         Sin d^V
88 *
89 ASIN
90 60
91 *
92 FIX 0
93 RND            L
94 RCL 15
95 X<=Y?          WP behind starting fix?
96 GTO 21
97 X<>Y
98 RCL 16
99 X>Y?           WP behind last WP?
100 GTO 21
101 RDN
102 ENTER↑        L
103 X<> 16        Last L
104 XEQ 03        Output leg data
105 ADV
106 FIX 1
107 CLA
108 ARCL 05
109 XEQ 15        Output WP ident.
110 CLA
111 RCL 07
112 XEQ 10        Azimuth format
113 "⊢, "
114 RCL 10        |α^WP|
115 SIN
116 RCL 09        Sin d^V
117 *
118 ASIN
119 60
120 *             WP radial
121 ARCL X
122 "⊢ NM"
```

```
LBL'RNAY
END          867 BYTES
```

```
123 XEQ 15          Output WP data
124 RCL 08
125 STO 04          Var update
126 ADV
127 RCL 05
128 STO 01
129 GTO 21
130♦LBL 02
131 CF 06
132 RCL 11
133 STO 05
134 RCL 14
135 STO 08          Var update
136 RCL 15
137 RCL 16
138 XEQ 03          Output leg data
139 RCL 12
140 STO 02
141 RCL 13
142 STO 03          Update starting
143 RCL 08             coordinates.
144 STO 04
145 GTO 20
146♦LBL 03          LEG OUTPUT
147 FIX 0
148 -
149 CLA
150 ARCL 01
151 FC? 55
152 GTO 00
153 ACA
154 CLA
155 SF 13
156 " TO "          Identifiers
157 ACA
158 CF 13
159 CLA
160♦LBL 00
161 FC? 55
162 "⊢-"
163 ARCL 05
164 XEQ 15          Output
165 "MC="
166 RCL 00          Ø
167 RCL 04          Var'
168 +               MC'
169 STO 06
170 XEQ 10          Azimuth format
171 RDN
172 CLX
173 RCL 06          MC'
174 RCL 00          Ø
175 RCL 08          Var
176 +               MC
177 X≠Y?
178 "⊢/"
179 X≠Y?            Var changed?
180 XEQ 10          Azimuth format
181 XEQ 15          Output MC
182 "DIST="
183 ARCL T          Leg distance
184 "⊢ NM"
185 GTO 15          Output leg distance
186♦LBL 04          COORDINATE INPUT
187 8
188 FS? 05          Departure point?
189 4
190♦LBL 05
191 STO 09
192♦LBL 06
193 AON
194 TONE 8
195 PROMPT
196 AOFF
197 FC? 23          Input declined?
198 RTN
199 FC? 06
200 CF 08
201 ASTO 05
202 3               If identifier is
203 ALENG              three or four letters
204 X=Y?               than search "F" file,
205 GTO 01             and prgm memory.
206 4
207 X≠Y?
208 GTO 00
209♦LBL 01
210 SF 25
211 XEQ IND 05      Search prgm memory for
212 FS?C 25            fix program
213 GTO 07
214 0
215 SEEKPT
216 POSFL           Search "F" file
217 X<0?            Ident. found?
218 GTO 01
219 GETREC
220 GETREC
221 ANUM
222 GETREC
223 ANUM
224 GETREC
225 ANUM
226♦LBL 07          Store fix coordinates
227 STO IND 09
228 DSE 09
229 RDN
230 HR
231 STO IND 09
232 DSE 09
233 RDN
234 HR
235 STO IND 09
236 RTN
237♦LBL 01          Prompt for coordinates
238 CF 22
239 "⊢ LAT?"
240 TONE 8
241 PROMPT
242 CLA
243 ARCL 05
244 FC?C 22
```

```
LBL'RNAV
END          867 BYTES
```

```
245 GTO 00
246 "⊢ LNG?"
247 TONE 8
248 PROMPT
249 CLA
250 ARCL 05
251 FC?C 22
252 GTO 00
253 "⊢ VAR?"
254 TONE 8
255 PROMPT
256 FS? 22
257 GTO 07
258*LBL 00          INTERSECTION
259 FC? 06          WP or Reference fix?
260 FS? 07
261 GTO 06
262 RCL 02
263 STO 06          Prior fix coordinates
264 RCL 03            to be used for the
265 STO 07            reference  fix.
266 RCL 04
267 STO 08
268 RCL 05
269 STO 11
270 SF 07           Reference fix flag
271 CF 23
272 8
273 "NEED FIX?"
274 XEQ 05
275 CF 07
276 RCL 01
277 FS? 23          Reference fix used?
278 RCL 05
279 STO 10
280 RCL 11
281 STO 05
282 CF 22
283 "RADIAL?"       Prompt for Ø mc
284 TONE 8
285 PROMPT
286 CLA
287 ARCL 05
288 SF 23
289 FC?C 22
290 GTO 04
291 "DME?"          Prompt for distance
292 TONE 8
293 PROMPT
294 CLA
295 ARCL 05
296 FC? 22
297 GTO 04
298 X<>Y
299 FC? 55
300 GTO 00
301 ADV
302 "⊢ * WP:"
303 ARCL 10
304 PRA             Output Idents.
305 CLA
```

```
306 FIX 1
307 XEQ 10          Output intersection data
308 RDN
309*LBL 00
310 ", "
311 ARCL Y
312 "⊢ NM"
313 FS? 55
314 XEQ 15
315 RCL 08
316 FS? 05
317 STO 04
318 -
319 STO 09
320 RDN
321 60
322 /               d
323 SIN             Sin d
324 STO 10
325 RCL 09          Ø
326 SIN
327 *
328 ASIN
329 RCL 09
330 COS
331 RCL 10
332 *
333 ASIN
334 RCL 06          Lat'
335 +
336 STO 06          Lat
337 FS? 05          Departure fix?
338 STO 02
339 LASTX
340 +
341 2
342 /
343 COS
344 /
345 ST- 07
346 RCL 07          Lng
347 FS? 05
348 STO 03
349 SF 08
350 RTN
351*LBL 08          TC & DISTANCE
352 RCL 06
353 SIN
354 RCL 02
355 SIN
356 *               Sin Lat Sin Lat'
357 STO 09
358 RCL 06
359 COS
360 RCL 02
361 COS
362 *               Cos Lat Cos Lat'
363 STO 10
364 +
365 ACOS
366 SIN             Sin a
```

```
                LBL'RNAV
                END          867 BYTES
```

```
367 RCL 06
368 RCL 02
369 -              + Northerly
370 SIGN           - Southerly
371 *
372 RCL 10
373 RCL 07
374 RCL 03
375 -
376 X<0?
377 SF 05          Easterly
378 COS
379 *
380 RCL 09
381 +
382 ACOS           d
383 SIN
384 STO 09         Sin d
385 LASTX
386 60
387 *
388 STO 10         Distance
389 RDN
390 /
391 ASIN           α
392 FS? 05
393 CHS
394 90
395 +
396 180
397 FS?C 05
398 CLX
399 +              ø
400 RTN
401*LBL 09         TERMINAL WAYPOINT
402 RCL 05
403 STO 01
404 CLA
405 ARCL 05
406 "├ WP?"        Prompt for WP
407 SF 06
408 CF 23
409 XEQ 04
410 CF 06
411 FC? 23         WP declined?
412 RTN
413 ADV
414 "WP:"
415 ARCL 05
416 ASTO 05
417 CLA
418 ARCL 01
419 "├ * "
420 ARCL 05
421 XEQ 15         Output ident
422 CLA
423 RCL 01
424 STO 05
425 XEQ 08         ø^V
426 180
427 +              θ^V
428 RCL 08         Var^V
429 +
430 FIX 1
431 XEQ 10         Format azimuth
432 "├, "
433 ARCL 10        d^V
434 "├ NM"
435 GTO 15
436*LBL 10         DEGREE FORMAT
437 RND
438 360
439 MOD
440 X≠0?
441 GTO 00
442 LASTX
443 +
444*LBL 00
445 100
446 X>Y?
447 "├0"
448 SQRT
449 X>Y?
450 "├0"
451 ARCL Y
452 FC? 55
453 RTN
454 ACA
455 "a"D"          Degree Character =
456 ASTO X           $ F4 61 22 44 86
457 ACSPEC
458 CLA
459 RTN
460*LBL 22         DESTINATION
461 FC?C 08        Fix a WP?
462 XEQ 09         Prompt for terminal WP
463 ADV
464 CLA
465 ARCL 18
466 "├ TO "        Terminal identifiers
467 ARCL 11
468 AVIEW
469 FIX 0
470 "T/DIST "
471 ARCL 17        Total trip distance
472 FS? 55
473 "├ NM"
474*LBL 15         Output
475 FC? 55
476 GTO 00
477 ACA
478 PRBUF
479 RTN
480 GTO 01
481*LBL 00
482 TONE 8
483 AVIEW
484 END
```

DATA REGISTERS

00	True course		
01	Ident'		
02	Lat'		
03	Lng'		
04	Var'		
05	Ident		
06	Lat		
07	Lng		
08	Var		
09	scratch		
10	scratch		
11	Ident^{S}		
12	Lat^{S}		
13	Lng^{S}		
14	Var^{S}		
15	Leg distance		
16	WP distance		
17	Total distance		
18	Departure ident		

STATUS

SIZE 19 TOT. REG. 133 USER MODE

ENG ____ FIX ____ SCI ____ ON X OFF ____

DEG X RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
05	S	Departure fix	
		Easterly course	Westerly course
		$\alpha^{WP} < 0$	$\alpha^{WP} < 0$
06	C	Computing a WP	
07	C	Reference fix	
08	C	Terminal fix is a	WP

ASSIGNMENTS

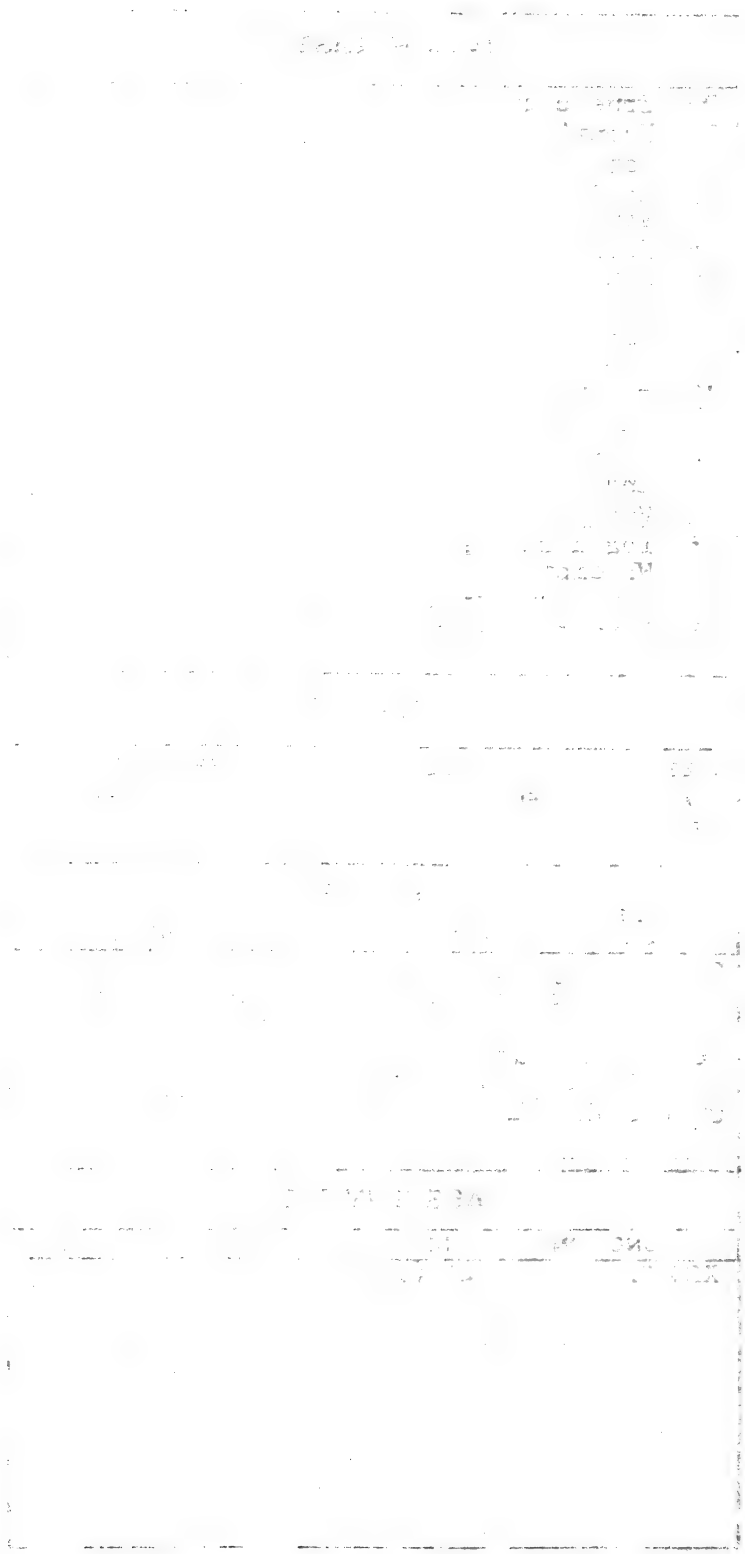
FUNCTION	KEY	FUNCTION	KEY
XEQ "NAV"	25 (TAN)		

```
LBL'RNAV
END          867 BYTES
```

by Robert A. TIMS

Sample Problem

```
PBF: * WP:PBF
182.2°, 4.4 NM

HILLE * WP:PBF
034.0°, 66.0 NM

PBF: to WP:LIT
MC=032°/031°
DIST=17 NM

WP:LIT
121.1°, 27.6 NM

WP:LIT to HILLE
MC=031°/032°
DIST=53 NM

HILLE to JBR
MC=023°
DIST=48 NM

JBR * WP:ARG
134.3°, 22.4 NM

PBF: TO JBR
T/DIST 118 NM
```



```
LBL'TAS
END                    177 BYTES
```

This program converts calibrated airspeed (CAS) to mach number (M) and true airspeed (TAS). Inputs required are pressure altitude (PALT), aircraft recovery coefficient (C_T) and indicated air temperature (IAT). Values for recovery coefficient vary from 0.6 to 1.0, but 0.8 is a good value for most aircraft.

The formulas used are less accurate for mach numbers above 1.0 (i.e., supersonic flight).

$$\text{Pressure ratio}\left(\frac{P}{P_o}\right) = \left[\frac{518.67 - 3.566 \times .10^{-3}\ \text{PALT}}{518.67}\right]^{5.2563}$$

$$M^2 = 5\left[\left(\frac{P_0}{P}\left\{\left[1 + 0.2\left(\frac{CAS}{661.5}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{0.2857} - 1\right]$$

$$TAS = 38.96M\sqrt{(IAT + 273)\left[C_T\left(\frac{1}{(1 + 0.2\,M^2)} - 1\right) + 1\right]}$$

STEP	INSTRUCTIONS	INPUT	FUNCTION	SIZE: 008 DISPLAY
1	To compute mach number and true airspeed		XEQ TAS	PALT= (ALT)?
2	Input pressure altitude.	PALT, ft.	R/S	CAS= (CAS)?
3	Input calibrated airspeed.	CAS, kt.	R/S	CT= (CT)?
4	Input aircraft recovery coefficient.	C_T	R/S	IAT= (IAT)?
5	Input indicated air temperature.	IAT, °C	R/S	MACH M TAS= (TAS)

```
01♦LBL "TAS"
02 SF 21
03 FIX 0
04 RCL 01
05 "PALT="
06 ARCL X
07 "┣?"
08 PROMPT
09 STO 01
10 RCL 02
11 "CAS="
12 ARCL X
13 "┣?"
14 PROMPT
15 STO 02
16 RCL 03
17 "IAT="
18 ARCL X
19 "┣?"
20 PROMPT
21 STO 03
22 FIX 1
23 RCL 04
24 "CT="
25 ARCL X
26 "┣?"
27 PROMPT
28 STO 04
29 RCL 01
30 -3566 E-6
31 *
32 518.67
33 +
34 LASTX
35 /
36 5.2563
37 Y↑X
38 STO 05
39 RCL 02
40 661.5
41 /
42 X↑2
43 5
44 /
45 1
46 +
47 3.5
48 Y↑X
49 1
50 -
51 RCL 05
52 /
53 1
54 +
55 3.5
56 1/X
57 Y↑X
58 1
59 -
60 5
61 *
62 SQRT
63 STO 06
64 FIX 2
65 "MACH "
66 ARCL X
67 AVIEW
68 RCL 06
69 X↑2
70 5
71 /
72 1
73 +
74 1/X
75 1
76 -
77 RCL 04
78 *
79 1
80 +
81 273
82 RCL 03
83 +
84 *
85 SQRT
86 RCL 06
87 *
88 38.96
89 *
90 FIX 0
91 "TAS="
92 ARCL X
93 AVIEW
94 END
```

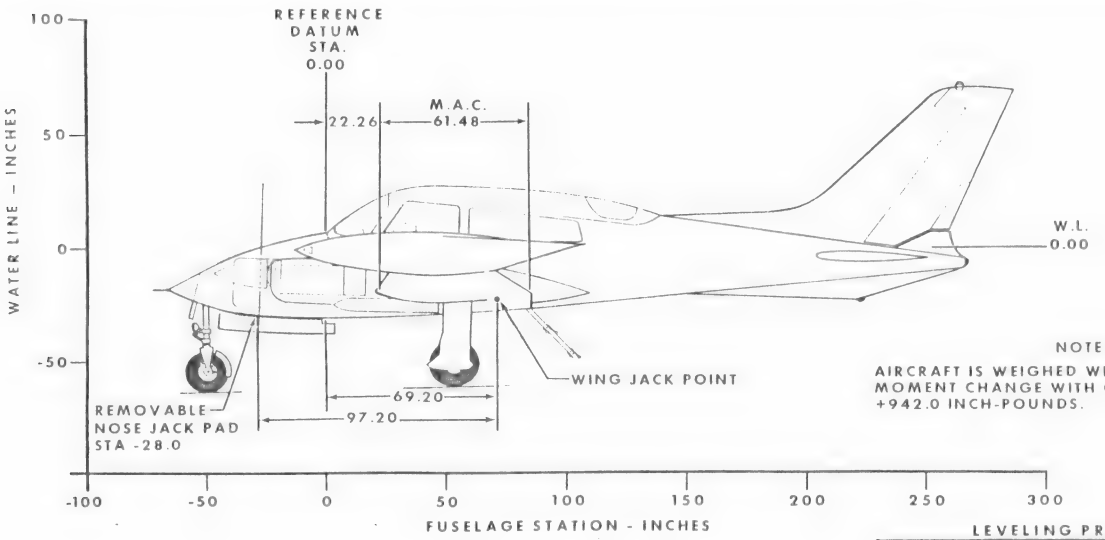
CESSNA AIRCRAFT COMPANY / WALLACE DIVISION / WICHITA KANSAS 67201

WEIGHT & BALANCE DATA

310Q

SERIAL NO. 310Q0601 AND ON

SERIAL NUMBER 310Q0726 REGISTRATION NUMBER N4155Q DATE 4-4-73



NOTE

AIRCRAFT IS WEIGHED WITH GEAR EXTENDED. MOMENT CHANGE WITH GEAR RETRACTED IS +942.0 INCH-POUNDS.

LEVELING PROVISIONS

LONGITUDINAL - LEFT SIDE OF FUSELAGE @ STA 59.10 & 88.90

LATERAL - BOTTOM OF FUSELAGE @ STA 37.5

computed

AIRCRAFT AS WEIGHED

POSITION	SCALE READING	SCALE DRIFT	TARE	NET WEIGHT
LEFT WING				
RIGHT WING				
NOSE				
AIRCRAFT TOTAL AS WEIGHED				

CG ARM OF AIRCRAFT AS WEIGHED = (69.2) − (97.2) × (NOSE NET WEIGHT) / (TOTAL AS WEIGHED) = ( ) INCHES AFT OF DATUM

NOTE

It is the responsibility of the pilot to insure that the aircraft is loaded properly. The licensed empty weight, CG and useful load for this aircraft as delivered from the factory is shown below. If the aircraft has been altered, refer to the latest weight and balance data for this information.

USEFUL LOAD

LICENSED GROSS WEIGHT	5300 LBS
LICENSED EMPTY WEIGHT	3552
USEFUL LOAD	1748

LICENSED EMPTY WEIGHT AND C G

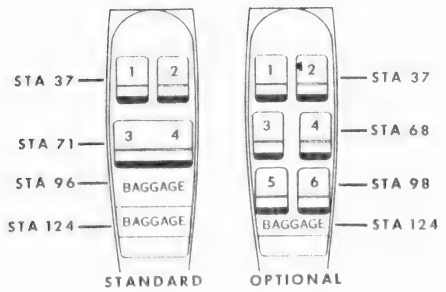
ITEM		WEIGHT (LBS)	CG ARM (IN)	MOMENT (IN-LBS)
AIRCRAFT (CALCULATED)		3485	38.4	133,855.
UNDRAINABLE FLUIDS		18.0	13.9	250
DRAINABLE UNUSABLE FUEL @ 6 LBS PER GALLON	TIP MAIN	12.0	44.0	528
	WING AUXILIARY	6.0	47.0	282
	WING LOCKER LEFT	[illegible]	[illegible]	[illegible]
	WING LOCKER RIGHT	[illegible]	[illegible]	[illegible]
PAINT		31.0	43.2	1323
OPTIONAL EQUIPMENT				
LICENSED EMPTY WEIGHT		3552	38.4	136,238.

Amended - 5-25-73
3610 38.08

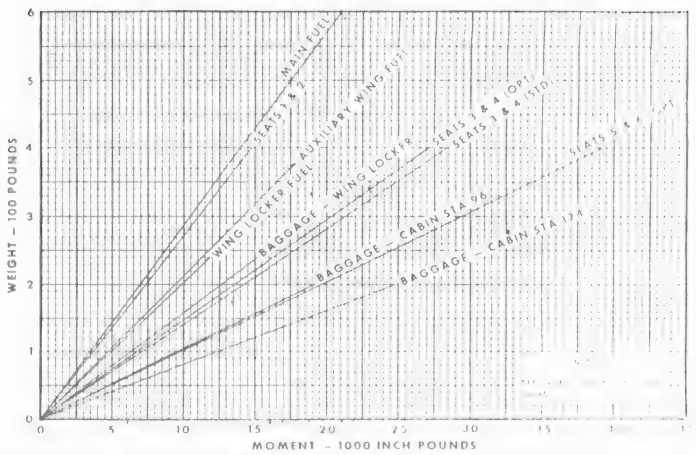
FORM NO. 1412-3 NOV 72

137453

PASSENGER SEAT AND BAGGAGE BAY MOMENT ARMS



LOADING MOMENTS CHART



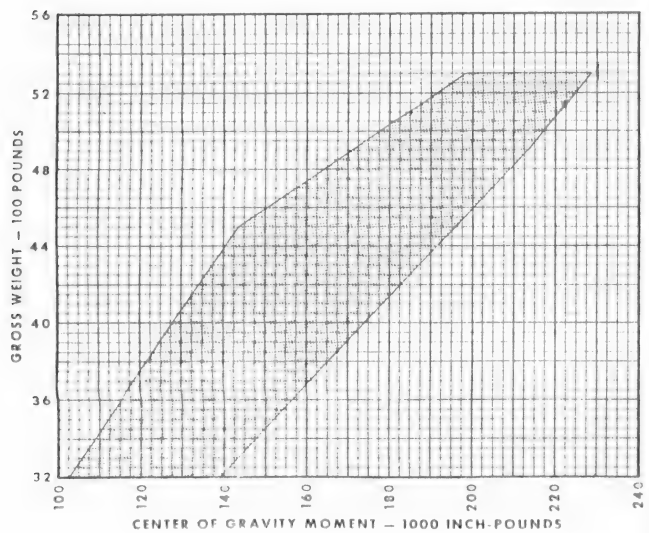
The following information is provided to enable a check of the aircraft load to determine if it falls within the approved center of gravity limits.

1) Determine weights, then obtain moments of useful load items from Loading Moments Chart, or multiply weight times CG arm.
2) Enter these weights and moments in the Loading Manifest Chart and total.
3) Enter the Center of Gravity Moment Envelope - Inch Pounds chart with the takeoff weight and center of gravity moment in inch pounds. If the intersection of these points falls within the envelope, the load is approved.
4) If desired, the Center of Gravity Moment Envelope - Inches chart may be used to determine whether the load is approved. CG arm is equal to the total moment divided by the total weight. Enter the chart with the takeoff weight and CG arm. If the intersection of these points fall within the envelope, the load is approved.

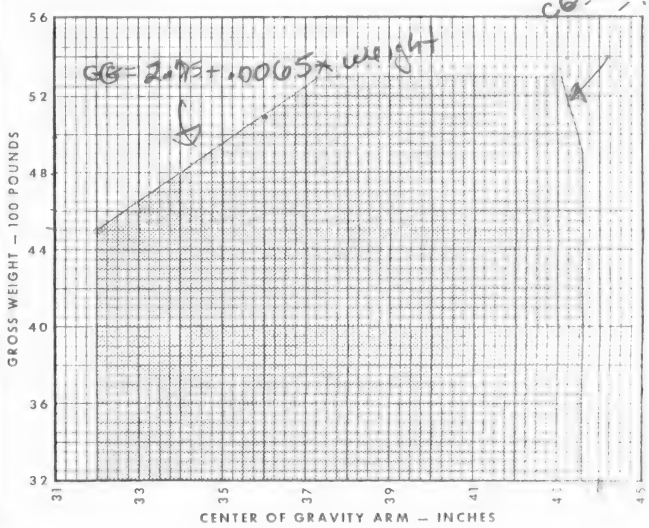
LOADING MANIFEST CHART

ITEM		WEIGHT (LBS)	CG ARM (IN)	MOMENT (1000 IN-LBS)
LICENSED EMPTY WEIGHT		3552	38.4	136.2
ALLOWABLE USEFUL LOAD 1748 LBS.	PILOT & PASSENGER SEATS 1 & 2	340	37	12.6
	PASSENGER SEATS 3 & 4	340	68	23.1
	PASSENGER SEATS 5 & 6	280	98	27.4
	BAGGAGE – CABIN STA 96		96	
	– CABIN STA 124		124	
	– WING LOCKER	23	63	1.4
	OIL (24 QT INCLUDING UNUSABLE)	45	-3.5	-0.2
	FUEL – TIP MAIN	600	35	21.0
	– WING AUXILIARY	120	47	5.6
	– WING LOCKER		49	
TAKEOFF GROSS WEIGHT		5300	42.8	227.1
SUBTRACT ALL FUEL INCLUDED ABOVE	FUEL – TIP MAIN	600	35	21.0
	– WING AUXILIARY	120	47	5.6
	– WING LOCKER		49	
ADD MINIMUM FUEL – TIP MAIN		133	35	4.7
ESTIMATED LANDING WEIGHT		4713	43.5	205.2

CG MOMENT ENVELOPE—INCH POUNDS



CG MOMENT ENVELOPE—INCHES



```
LBL'WB
END          401 BYTES
```

C310 N4155Q

```
PILOTS= 200
ROW 2= 300
ROW 3= 0
AFT BAG= 50
WING BAG= 100
MAIN FUEL= 100
AUX FUEL= 0
T/FUEL= 100
CG LIMITS: 34.63 43.48
CG= 40.49
GW= 4905 LBS
```

```
01♦LBL "WB"
02 12
03 PSIZE
04 CLRG
05 ADV
06 FIX 0
07 "C310 N4155Q"
08 SF 12
09 AVIEW
10 CF 12
11 SF 21
12 SF 22
13 CF 29
14 ADV          ARM moments
15 37
16 STO 00       PILOTS
17 68
18 STO 01       ROW 2
19 98
20 STO 02       ROW 3
21 124
22 STO 03       BAGGAGE
23 63
24 STO 04       wing lockers
25 35
26 STO 05       TIP TANKS
27 47
28 STO 06       AUX TANKS
29 CLX
30 "PILOTS"
31 XEQ 06
32 "ROW 2"
33 XEQ 06
34 "ROW 3"
35 CF 22
36 XEQ 06
37 "AFT BAG"
38 XEQ 06
39 "WING BAG"
40 XEQ 06
41 163
42 1645
43 RCL 08
44 -
45 6
46 /
47 "FUEL<= "
48 ARCL X
49 X<Y?
50 AVIEW
51 CLX
52 "MAIN FUEL"
53 XEQ 05
54 "AUX FUEL"
55 XEQ 05
56 "T/FUEL= "
57 ARCL 10
58 AVIEW
59 3655         Gross wt.
60 ST+ 08
61 37.56        Empty C.G.
62 *
63 RCL 09       TOTAL MOMENTS
64 +
65 RCL 08
66 /
67 STO 09       C.G.
68 4500
69 RCL 08
70 X>Y?
71 GTO 01
72 32
73 GTO 02
74♦LBL 01
75 .0065        FWD C.G.
76 *
77 2.75
78 +
79♦LBL 02
80 STO 07       FWD C.G
81 4900
82 RCL 08
83 X>Y?
84 GTO 03
85 43.48
86 GTO 04
87♦LBL 03
88 -.000775     AFT C.G.
89 *
90 47.2775
91 +
92♦LBL 04
93 FIX 2
94 "CG LIMITS: "
95 ARCL 07
96 "├ "
97 ARCL X
98 FC? 55
99 AVIEW
100 FS? 55
101 PRA
102 RCL 09
103 CLA
104 X>Y?
105 "AFT "
106 RCL 07
107 X>Y?
108 "FWD "
109 "├CG= "
110 ARCL Y
111 AVIEW
112 GTO 10
113♦LBL 05
114 "├= "       FUEL Weight
115 PROMPT
116 ST+ 10
117 ARCL X
118 FS? 55
119 PRA
120 6
121 *
122 GTO 07
123♦LBL 06
124 "├= "       ADD weight
125 PROMPT
126 ARCL X
127 FS? 55
128 PRA
129♦LBL 07
130 ST+ 08      TOTAL WT
131♦LBL 08
132 RCL IND 11
133 X<>Y
134 *
135 ST+ 09      MOMENTS
136 1
137 ST+ 11
138 CLX
139 RTN
140♦LBL 09
141 -
142 "OVER WT "
143 ARCL X
144 AVIEW
145 RTN
146♦LBL 10
147 FIX 0
148 "GW= "
149 ARCL 08
150 "├ LBS"
151 AVIEW
152 RCL 08
153 5300
154 X<Y?
155 XEQ 09
156 ADV
157 ADV
158 ADV
159 ADV
160 ADV
161 END
```

WIND CORRECTION

W I N D C O R R E C T I O N H P - 4 1 C P R O G R A M

Note: All velocities are absolute values.

True Heading:

$$TH=TC+SIN^{-1}[Wv*SIN(Wd-TC)/TAS]$$

Wv=Wind Velocity
Wd=Wind Direction

Ground Speed:

$$GS=[(TAS*SIN(TH)-Wv*SIN(Wd))^2+(TAS*COS(TH)-Wv*COS(Wd))^2]^{1/2}$$

If COS(Wd-TH) is negative and TAS<Wv then the GS is negative.

Wind Direction:

$$Wd=TAN^{-1}[(TAS*SIN(TH)-GS*SIN(TC))/(TAS*COS(TH)-GS*COS(TC))]$$

If TAS*COS(TH)-GS*COS(TC) is negative then add 180 to the Wd.

Wind Velocity:

$$Wv=[(TAS*SIN(TH)-GS*SIN(TC))^2+(TAS*COS(TH)-GS*COS(TC))^2]^{1/2}$$

Data Registers:

R00 - Time in HR format
R01 - Wd
R02 - Wv
R03 - TC
R04 - TAS
R05 - TH
R06 - GS
R07 - Distance

```
LBL"WCA
END          591 BYTES
```

```
01*LBL "WCA"
02 8
03 PSIZE
04 FIX 0
05 CF 29
06 CLRG
07*LBL 16
08 0
09 X<>F
10 CF 22
11 "GS/TH   WIND"
12 PROMPT
13*LBL A    GS + TH
14 ADV
15 1
16 "WIND DIR=" Get
17 XEQ 20      Wind
18 2
19 "WIND VEL="
20 XEQ 23   Get
21 XEQ 17   TC + TAS
22 RCL 01
23 RCL 03
24 -
25 SIN
26 RCL 02      WCA
27 *
28 RCL 04
29 /
30 ASIN
31 RCL 03   TC
32 +
33 XEQ 21   DEGREE MOD
34 STO 05   TH
35 SIN
36 RCL 04
37 *
38 RCL 01
39 SIN
40 RCL 02
41 *
42 -
43 X↑2
44 RCL 05      GS
45 COS
46 RCL 04
47 *
48 RCL 01
49 COS
50 RCL 02
51 *
52 -
53 X↑2
```

```
54 +
55 SQRT
56 STO 06
57 RCL 01
58 RCL 05      GS
59 -           DIRECTION
60 COS
61 X<0?
62 GTO 01
63 RCL 04
64 RCL 02
65 -
66 SIGN
67 ST* 06
68*LBL 01
69 ADV
70 5           OUTPUT
71 "TH="       TH + GS
72 SF 01
73 FS? 55
74 SF 02
75 XEQ 20
76 6
77 "GS="
78 SF 01
79 FS? 55
80 SF 02
81 XEQ 23
82 ADV
83 XEQ 18   Get DISTANCE
84 RCL 07
85 RCL 06
86 /
87 HMS
88 "TIME="  OUTPUT
89 FS? 55   TIME enRoute
90 GTO 02
91 FIX 4
92 ARCL X
93 FIX 0
94 PROMPT
95 GTO C
96*LBL 02
97 XEQ 19
98*LBL C       PAPER OUT
99 ADV
100 ADV
101 ADV
102 ADV
103 ADV
104 GTO 16
105*LBL E    WIND
106 ADV
```

```
107 "TIME=" INPUT Time
108 PROMPT  enRoute
109 FC?C 22 - IF INPUT
110 GTO 03    declined
111 HR          Goto
112 STO 00      GS INPUT
113 LASTX
114 FS? 55
115 XEQ 19  PRINT Time
116 XEQ 18  Get Distance
117 RCL 07
118 RCL 00
119 /
120 STO 06  G.S.
121 ADV
122 FS? 55
123 SF 02
124*LBL 03
125 6       Get or Output
126 "GS="   GS
127 XEQ 23
128 5       Get
129 "TH="   TH
130 XEQ 20
131 XEQ 17  Get TC + TAS
132 180  - used if direction
                is southerly
133 RCL 05
134 SIN
135 RCL 04
136 *
137 RCL 03
138 SIN
139 RCL 06      Wind
140 *           DIRECTION
141 -
142 RCL 05
143 COS
144 RCL 04
145 *
146 RCL 03
147 COS
148 RCL 06
149 *
150 -
151 X<0?    TEST FOR
152 SF 01   DIRECTION
153 /
154 ATAN
155 FS?C 01
156 +
157 XEQ 21  DEGREE MOD
158 STO 01
159 RCL 05
```

```
LBL'WCA
END                    591 BYTES
```

```
160 SIN
161 RCL 04
162 *
163 RCL 03
164 SIN
165 RCL 06
166 *
167 -
168 X↑2
169 RCL 05
170 COS
171 RCL 04
172 *
173 RCL 03
174 COS
175 RCL 06
176 *
177 -
178 X↑2
179 +
180 SQRT
181 STO 02
182 ADV
183 1
184 "WIND DIR="
185 SF 01
186 FS? 55
187 SF 02
188 XEQ 20
189 2
190 "WIND VEL="
191 SF 01
192 FS? 55
193 SF 02
194 XEQ 23
195 GTO C
196♦LBL 17
197 3
198 "TC="
199 XEQ 20
200 4
201 "TAS="
202 GTO 23
203♦LBL 18
204 FIX 1
205 RCL 07
206 RND
207 "DIST="
208 FS? 55
209 ACA
210 ARCL X
211 PROMPT
212 FS?C 22
```

wind velocity

output wind

INPUT TC & TAS

INPUT DISTANCE

```
213 STO 07
214 FIX 0
215 FC? 55
216 RTN
217 FIX 1
218 RND
219 ACX
220 FIX 0
221 " NM"
222 ACA
223 PRBUF
224 RTN
225♦LBL 19
226 ACA
227 INT
228 ACX
229 " H"
230 114
231 XTOA
232 ACA
233 LASTX
234 FRC
235 100
236 *
237 INT
238 ACX
239 " M"
240 105
241 XTOA
242 110
243 XTOA
244 ACA
245 LASTX
246 FRC
247 100
248 *
249 RND
250 ACX
251 " S"
252 101
253 XTOA
254 99
255 XTOA
256 ACA
257 PRBUF
258 RTN
259♦LBL 20
260 RCL IND X
261 RND
262 XEQ 22
263 FS? 55
264 ACA
265 ARCL X
```

Print TIME

INPUT Degree or Direction

```
266 FC?C 02
267 PROMPT
268 XEQ 21
269 FS?C 01
270 GTO 04
271 FS?C 22
272 STO IND Z
273♦LBL 04
274 FC? 55
275 RTN
276 RND
277 XEQ 22
278 ACX
279 "a"D"
280 ASTO X
281 ACSPEC
282 PRBUF
283 RTN
284♦LBL 21
285 360
286 MOD
287♦LBL 22
288 X>0?
289 RTN
290 360
291 +
292 RTN
293♦LBL 23
294 RCL IND X
295 RND
296 FS? 55
297 ACA
298 ARCL X
299 FC?C 02
300 PROMPT
301 FS?C 01
302 GTO 05
303 ABS
304 FS?C 22
305 STO IND Z
306♦LBL 05
307 FC? 55
308 RTN
309 RND
310 ACX
311 CLA
312 " K"
313 116
314 XTOA
315 115
316 XTOA
317 ACA
318 PRBUF
319 END
```

Degree MOD

$0° = 360°$

INPUT Speed

CAP GRID SEARCH

CAP GRID SEARCH HP-41C PROGRAM

SAMPLE PRINTOUT
2 legs instead of 8

Data Registers:

R00 - True Course
R01 - Ident'
R02 - Lat'
R03 - Lng'
R04 - Var'
R05 - Ident
R06 - Lat
R07 - Lng
R08 - Var
R09 - Scratch
R10 - Scratch
R11 - Total Distance
R12 - Grid Leg 1b Lat/Lng
R13 - # of Legs
R14 - Exit Lat/Lng
R14 - Lat/Lng Increment
R15 - 1st Waypoint Ident
R16 - 2nd Waypoint Ident

Flags:

F00 - Computing the Grid
F01 - East/West Search
F02 - Compute End of Leg, Point b
F03 - Compute Exit Waypoints
F04 - Initialize Grid Parameters
F05 - Base Fix
F05 - Easterly Course

Note: Lat & Lng are in degrees & tenths.
Fix coordinates are given in program form.
Grid parameters are given in program form.
(See example given on last page)

```
JBR
  ARG: 134.3°, 22.4 NM

TC=244°
MC=240°
DIST=28.7 NM

FIX
  ARG: 196.6°, 31.1 NM
  GQE: 292.6°, 38.1 NM

TC=090°
MC=086°
DIST=8.7 NM

ENTRY
  ARG: 180.5°, 29.2 NM
  GQE: 299.2°, 30.5 NM

TC=090°
MC=086°
DIST=12.2 NM

LEG 1b
  ARG: 157.2°, 30.7 NM
  GQE: 317.5°, 21.3 NM

TC=180°
MC=176°
DIST=7.0 NM

LEG 2a
  ARG: 160.7°, 37.4 NM
  GQE: 302.1°, 16.4 NM

TC=270°
MC=266°
DIST=12.2 NM

EXIT
  ARG: 179.6°, 36.2 NM
  GQE: 286.8°, 27.3 NM

TC=042°
MC=038°
DIST=25.9 NM

JBR
  GQE: 340.2°, 30.2 NM

T/DIST=94.7 NM
```

```
01♦LBL "GRID"
02 17
03 PSIZE
04♦LBL 30
05 32
06 X<>F
07 SF 21
08 CF 22
09 CF 23
10 CF 29
11 0
12 STO 11
13 "BASE?"
14 XEQ 03
15 FC?C 23
16 GTO 30
17♦LBL 01
18 RCL 05
19 STO 01
20 ADV
21 CLA
22 ARCL 01
23 FC? 55
24 GTO 02
25 SF 12
26 PRA
27 CF 12
28♦LBL 02
29 FS? 00
30 GTO 15
31 FS? 03
32 GTO 16
33 XEQ 09
34 FS?C 23
35 XEQ 09
36♦LBL 20
37 CLA
38 ARCL 01
39 "⊢ TO?"
40 XEQ 03
41 FS?C 23
42 GTO 21
43 ADV
44 "T/DIST="
45 ARCL 11
46 FS? 55
47 "⊢ NM"
48 XEQ 11
49 ADV
50 ADV
51 ADV
52 ADV
53 ADV
54 GTO 30
55♦LBL 21
56 ADV
57 XEQ 08
58 STO 00
59 RCL 10
60 FIX 1
```

SET FLAGS CLEAR OTHERS

GET BASE

COMPUTE GRID LEG

EXIT WAYPOINTS

FIND WAYPOINTS

GET FIX

ENDING

PAPER OUT

```
61 RND
62 ST+ 11
63 STO 10
64 FIX 0
65 "TC="
66 RCL 00
67 XEQ 13
68 XEQ 11
69 "MC="
70 RCL 00
71 RCL 04
72 +
73 STO 09
74 XEQ 13
75 CLX
76 RCL 09
77 RCL 00
78 RCL 08
79 +
80 X≠Y?
81 "⊢/"
82 X≠Y?
83 XEQ 13
84 XEQ 11
85 FIX 1
86 "DIST="
87 ARCL 10
88 "⊢ NM"
89 XEQ 11
90 RCL 06
91 STO 02
92 RCL 07
93 STO 03
94 RCL 08
95 STO 04
96 GTO 01
97♦LBL 03
98 8
99 FS?C 05
100 4
101 STO 09
102♦LBL 04
103 AON
104 TONE 8
105 PROMPT
106 AOFF
107 FC? 23
108 RTN
109 ASTO 05
110 SF 25
111 XEQ IND 05
112 FC?C 25
113 GTO 07
114♦LBL 05
115 STO IND 09
116 DSE 09
117 RDN
118 XEQ 06
119 STO IND 09
120 DSE 09
```

Print TC MC Dist Leg

GET FIX COORDINATES

```
121 RDN
122 XEQ 06
123 STO IND 09
124 RTN
125♦LBL 06
126 100
127 *
128 HMS
129 100
130 /
131 HR
132 RTN
133♦LBL 07
134 "⊢ LAT?"
135 TONE 8
136 PROMPT
137 CLA
138 ARCL 05
139 FC?C 22
140 GTO 04
141 "⊢ LNG?"
142 TONE 8
143 PROMPT
144 CLA
145 ARCL 05
146 FC?C 22
147 GTO 04
148 "⊢ VAR?"
149 TONE 8
150 PROMPT
151 CLA
152 ARCL 05
153 FC?C 22
154 GTO 04
155 GTO 05
156♦LBL 08
157 RCL 06
158 SIN
159 RCL 02
160 SIN
161 *
162 STO 09
163 RCL 06
164 COS
165 RCL 02
166 COS
167 *
168 STO 10
169 +
170 ACOS
171 SIN
172 RCL 06
173 RCL 02
174 -
175 SIGN
176 *
177 RCL 10
178 RCL 07
179 RCL 03
180 -
```

CALCULATE TC + LEG DISTANCE

```
181 X<0?
182 SF 05
183 COS
184 *
185 RCL 09
186 +
187 ACOS
188 SIN
189 STO 09
190 LASTX
191 60
192 *
193 STO 10
194 RDN
195 /
196 ASIN
197 FS? 05
198 CHS
199 90
200 +
201 180
202 FS?C 05
203 CLX
204 +
205 RTN
206*LBL 09
207 CLA
208 ARCL 01
209 "⊢ WP?"
210 XEQ 03
211 FC? 23
212 RTN
213*LBL 10
214 CLA
215 FS? 55
216 "  "
217 ARCL 05
218 "⊢: "
219 XEQ 08
220 180
221 +
222 RCL 08
223 +
224 FIX 1
225 XEQ 13
226 "⊢, "
227 ARCL 10
228 FS? 55
229 "⊢ NM"
230*LBL 11
231 FC? 55
232 GTO 12
233 ACA
234 PRBUF
235 RTN
236*LBL 12
237 TONE 8
238 AVIEW
239 RTN
240*LBL 13
241 RND
242 360
243 MOD
244 X≠0?
245 GTO 14
246 LASTX
247 +
248*LBL 14
249 100
250 X>Y?
251 "⊢0"
252 SQRT
253 X>Y?
254 "⊢0"
255 ARCL Y
256 FC? 55
257 RTN
258 ACA
259 "a"D"
260 ASTO X
261 ACSPEC
262 CLA
263 RTN
264*LBL 15
265 FC?C 04
266 GTO 16
267 RCL 12
268 XEQ 06
269 STO 12
270 3
271 FS? 01
272 2
273 RCL 14
274 XEQ 06
275 RCL IND Y
276 -
277 RCL 13
278 /
279 STO 14
280 1 E3
281 ST/ 13
282 SF 02
283*LBL 16
284 FC? 55
285 XEQ 12
286 RCL 15
287 STO 05
288 8
289 STO 09
290 SF 25
291 XEQ IND 15
292 FC? 25
293 GTO 00
294 XEQ 05
295 XEQ 10
296 RCL 16
297 STO 05
298 8
299 STO 09
300 XEQ IND 16
301 FC?C 25
302 GTO 00
303 XEQ 05
304 XEQ 10
305*LBL 00
306 FS?C 03
307 GTO 20
308 FIX 0
309 FC?C 02
310 GTO 00
311 CF 00
312 ISG 13
313 SF 00
314 "LEG "
315 ARCL 13
316 "⊢b"
317 FC? 00
318 "EXIT"
319 ASTO 05
320 RCL 04
321 STO 08
322 RCL 03
323 FS? 01
324 X<> 12
325 STO 07
326 RCL 02
327 FC? 01
328 X<> 12
329 STO 06
330 FC? 00
331 SF 03
332 GTO 21
333*LBL 00
334 SF 02
335 RCL 13
336 1
337 +
338 "LEG "
339 ARCL X
340 "⊢a"
341 ASTO 05
342 RCL 04
343 STO 09
344 RCL 14
345 RCL 03
346 FC? 01
347 +
348 STO 07
349 RCL 14
350 RCL 02
351 FS? 01
352 +
353 STO 06
354 GTO 21
355 END
```

GET WAYPOINT

OUT PUT WAYPOINT DATA

OUTPUT DATA

OUTPUT degrees

INITIALIZE GRID PARAMETERS

LEG INCREMENT

LEG COUNTER

CALCULATE WAYPOINTS

END GRID

LEG END

LEG START

```
01*LBL "45CD"
02 "LEG1 TC"
03 90
04 COS
05 X=0?
06 SF 01
07 "LEG1 END"
08 90.45
09 STO 12
10 "NO. LEGS"
11 8
12 1
13 -
14 STO 13
15 "EXIT"
16 35.305
17 91
18 FS? 01
19 X<>Y
20 STO 14
21 "WAYPOINTS"
22 "ARG"
23 ASTO 15
24 "GOE"
25 ASTO 16
26 "ENTRY"
27 ASTO 05
28 35.375
29 91
30 -4
31 SF 00
32 SF 04
33 RTN
34*LBL "ARG"
35 36.066
36 90.572
37 -4
38 RTN
39*LBL "GOE"
40 35.208
41 90.287
42 -4
43 RTN
44*LBL "JBR"
45 35.499
46 90.388
47 -4
48 RTN
49*LBL "FIX"
50 35.375
51 91.107
52 -4
53 END
```

A GRID 45 SECTORS C+D

SEARCH SAMPLE

— 1st Leg COURSE = 90°

— ENDING LNG (IF S/N SEARCH WOULD BE LAT)

— must be greater than 1

EXIT COORDINATES

— E/W SEARCH - STORES LNG
N/S SEARCH - STORES LAT

— 1st & 2nd WAYPOINTS

ENTRY COORDINATES - includes VAR

— compute the grid flag
— PARAMETER FLAG
— misc. fixes needed

— A fix west of 1st Leg Just South of Newport Airport

CURVE FITTING

CURVE FITTING

For a set of data points (x_i, y_i), i = 1, 2, ..., n, this program can be used to fit the data to any of the following curves:

1. Straight line (linear regression): $y = a + bx$.
2. Exponential curve: $y = ae^{bx}$ $(a > 0)$,
3. Logarithmic curve: $y = a + b \ln x$,
4. Power curve: $y = ax^b$ $(a > 0)$.

The regression coefficients a and b are found by solving the following equivalent system of linear equations.

$$An + B\Sigma X_i = \Sigma Y_i$$

$$A\Sigma X_i + B\Sigma X_i^2 = \Sigma Y_i X_i$$

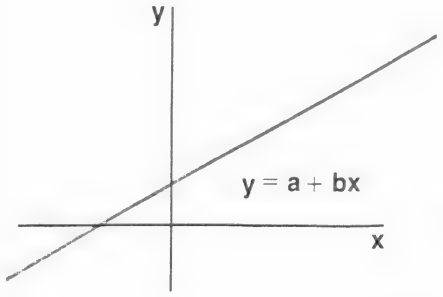
The relations of the variables are defined by the following:

Regression	A	B	X_i	Y_i
Linear	a	b	x_i	y_i
Exponential	ln a	b	x_i	$\ln y_i$
Logarithmic	a	b	$\ln x_i$	y_i
Power	ln a	b	$\ln x_i$	$\ln y_i$

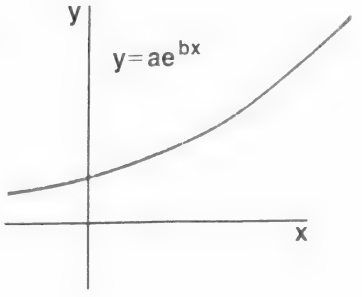
The coefficient of determination is:

$$R^2 = \frac{A\Sigma Y_i + b\Sigma X_i Y_i - \frac{1}{n}(\Sigma Y_i)^2}{\Sigma(Y_i^2) - \frac{1}{n}(\Sigma Y_i)^2}$$

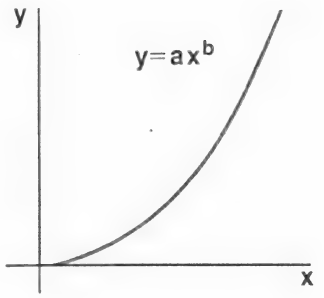
Linear Regression



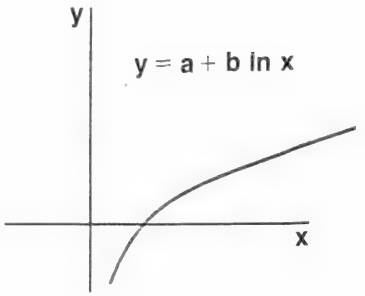
Exponential Curve Fit



Power Curve Fit



Logarithmic Curve Fit



Remarks:

1. The program applies the least square method, either to the original equations (straight line and logarithmic curve) or to the transformed equations (exponential curve and power curve).
2. Negative and zero values of x_i will cause a calculator error for logarithmic curve fits. Negative and zero values of y_i will cause a machine error for exponential curve fits. For power curve fits both x_i and y_i must be positive, non-zero values.
3. As the differences between x and/or y values become small, the accuracy of the regression coefficients will decrease.

SIZE: 016

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Set status and key in the program			
2	Initialize the program for STRAIGHT LINE or for EXPONENTIAL CURVE or for LOGARITHMIC CURVE or for POWER CURVE		 XEQ LIN XEQ EXP XEQ LOG XEQ POW	 LIN EXP LOG POW
3	Repeat step 3 and 4 for i=1,2,..., n input: x_i y_i	 x_i y_i	 ENTER↑ A	 (i)
4	If you made a mistake in inputting x_k and y_k, then correct by→	x_k y_k	ENTER↑ C	 (k−1)
5	Calculate R^2 and regression coefficients a and b		E R/S R/S	R2=(R^2) a=(a) b=(b)

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
6	Calculate estimated y from regression, input x	x	R/S	Y.= (ŷ)
7	Repeat step 6 for different x's			
8	Repeat step 5 if you want the results again			
9	To use the same program for another set of data, initialize the program by →		A	LIN or EXP or LOG or POW
	then go to step 3			
10	To use another program, go to step 2			

Example 1:

Fit a straight line to the following set of data and compute ŷ for x = 37 and x = 35.

x_i	40.5	38.6	37.9	36.2	35.1	34.6
y_i	104.5	102	100	97.5	95.5	94

Keystrokes:	Display:	
XEQ ALPHA LIN ALPHA	*LIN*	
40.5 ENTER↑ 104.5 A	*1.00*	
38.6 ENTER↑ 102 A	*2.00*	
37.9 ENTER↑ 100 A	*3.00*	
36.2 ENTER↑ 97.5 A	*4.00*	
35.2 ENTER↑ 95.5 A	*5.00*	Oops!
35.2 ENTER↑ 95.5 C	*4.00*	Correct error.
35.1 ENTER↑ 95.5 A	*5.00*	Use proper values.
34.6 ENTER↑ 94 A	*6.00*	
E	*R2 = 0.99*	
R/S	*a = 33.53*	
R/S	*b = 1.76*	
37 R/S	*Y. = 98.65*	
35 R/S	*Y. = 95.13*	

Example 2:

Fit an exponential curve to the following set of data and compute ŷ for x = 1.5 and x = 2.

x_i	.72	1.31	1.95	2.58	3.14
y_i	2.16	1.61	1.16	.85	0.5

Keystrokes:	Display:	
XEQ ALPHA EXP ALPHA	*EXP*	
.72 ENTER↑ 2.16 A	*1.00*	
1.31 ENTER↑ 1.61 A	*2.00*	
1.95 ENTER↑ 1.16 A	*3.00*	If you don't make a mistake you can skip two steps.
2.58 ENTER↑ .85 A	*4.00*	
3.15 ENTER↑ .05 A	*5.00*	
3.15 ENTER↑ .05 C	*4.00*	
3.14 ENTER↑ 0.5 A	*5.00*	
E	*R2 = 0.98*	
R/S	*a = 3.45*	
R/S	*b = −0.58*	
1.5 R/S	*Y. = 1.44*	
2.0 R/S	*Y. = 1.08*	

Example 3:

Fit a logarithmic curve to the following set of data and compute ŷ for x = 8 and x = 14.5.

x_i	3	4	6	10	12
y_i	1.5	9.3	23.4	45.8	60.1

Keystrokes:	Display:	
XEQ ALPHA LOG ALPHA	*LOG*	
3 ENTER↑ 1.5 A	*1.00*	
4 ENTER↑ 9.3 A	*2.00*	
6 ENTER↑ 23.4 A	*3.00*	
10 ENTER↑ 45.8 A	*4.00*	
12 ENTER↑ 6.01 A	*5.00*	Another mistake!
12 ENTER↑ 6.01 C	*4.00*	
12 ENTER↑ 60.1 A	*5.00*	
E	*R2 = 0.98*	
R/S	*a = −47.02*	
R/S	*b = 41.39*	
8 R/S	*Y. = 39.06*	
14.5 R/S	*Y. = 63.67*	

Example 4:

Fit a power curve to the following set of data and compute $\hat{y}$ for x = 18 and x = 23.

x_i	10	12	15	17	20	22	25	27	30	32	35
y_i	0.95	1.05	1.25	1.41	1.73	2.00	2.53	2.98	3.85	4.59	6.02

Keystrokes:	**Display:**	
XEQ ALPHA POW ALPHA	*POW*	
10 ENTER↑ 0.95 A	*1.00*	
12 ENTER↑ 1.05 A	*2.00*	
15 ENTER↑ 1.25 A	*3.00*	
17 ENTER↑ 1.41 A	*4.00*	
20 ENTER↑ 1.73 A	*5.00*	
22 ENTER↑ 2.00 A	*6.00*	
25 ENTER↑ 2.53 A	*7.00*	
27 ENTER↑ 2.98 A	*8.00*	
30 ENTER↑ 3.85 A	*9.00*	
32 ENTER↑ 4.59 A	*10.00*	
35 ENTER↑ 60.2 A	*11.00*	
35 ENTER↑ 60.2 C	*10.00*	Error correction again.
35 ENTER↑ 6.02 A	*11.00*	
E	*R2 = 0.94*	
R/S	*a = 0.03*	
R/S	*b = 1.46*	
18 R/S	*Y. = 1.76*	
23 R/S	*Y. = 2.52*	

Programming Highlight

This program uses a single section of code for most of the calculations it needs to do. Since each of the four types of curve fitting requires the input data to be in a different form, it would seem that a different program should be used for each curve type. Instead, each of the set-up programs, LIN, LOG, EXP, and POW, stores a code in register 00. Then the single function on line 32, XEQ IND 00, takes care of the four different ways of processing the input data by executing the function whose label is stored in register 00.


```
01♦LBL "LIN
02 5
03 "LIN"
04 GTO 13
05♦LBL "EXP
06 6
07 "EXP"
08 GTO 13
09♦LBL "LOG
10 7
11 "LOG"
12 GTO 13
13♦LBL "POW
14 8
15 "POW"
16♦LBL 13
17 XEQ "INI
T"
18 STO 00
19 ASTO 08
20 ΣREG 10
21 CLΣ
22 BEEP
23 AVIEW
24 STOP
25♦LBL C
26 X<>Y
27 XEQ IND
00
28 Σ-
29 STOP
30♦LBL A
31 X<>Y
32 XEQ IND
00
33 Σ+
34 STOP
35♦LBL 07
36 LN
37 RTN
38♦LBL 08
39 LN
40♦LBL 06
41 X<>Y
42 LN
43 X<>Y
44 RTN
```

Linear. Exponential. Logarithmic. Power. Beep, display and Σ registers. Correction. Input data. Log. Power and exp.

```
45♦LBL E
46 RCL 15
47 RCL 11
48 RCL 10
49 RCL 10
50 XEQ 09
51 STO 03
52 RCL 12
53 RCL 11
54 RCL 10
55 RCL 14
56 XEQ 09
57 RCL 03
58 -
59 STO 04
60 XEQ IND
00
61 STO 06
62 RCL 15
63 RCL 14
64 RCL 10
65 RCL 12
66 XEQ 09
67 RCL 03
68 -
69 STO 05
70♦LBL 03
71 RCL 04
72 RCL 12
73 +
74 RCL 05
75 RCL 14
76 +
77 +
78 RCL 12
79 X↑2
80 RCL 15
81 -
82 STO 09
83 -
84 RCL 13
85 RCL 09
86 -
87 -
88 "R2"
89 XEQ 88
90 RCL 06
91 "a"
92 XEQ 88
93 RCL 05
94 "b"
```

Calculate A, b and a, b.

prompt

prompt

R00 = Index
R01 = x
R02 = y
R03 = det
R04 = A
R05 = b
R06 = a
R07 = used
R08 = LIN or EXP or LOG or POW
R09 = (Σy) 2/n

```
 95 GTO 01
 96♦LBL 06
 97♦LBL 08
 98 E↑X
 99♦LBL 05
100♦LBL 07
101 RTN
102♦LBL 09
103 *
104 STO 07
105 RDN
106 *
107 RCL 07
108 -
109 RTN
110♦LBL 00
111 "Y."
112♦LBL 01
113 "⊢="
114 ARCL X
115 AVIEW
116 PSE 55
117 STOP
118♦LBL 04
119 GTO IND
00
120♦LBL 08
121 RCL 05
122 Y↑X
123 GTO 09
124♦LBL 06
125 RCL 05
126 *
127 E↑X
128♦LBL 09
129 RCL 06
130 *
131 GTO 00
132♦LBL 07
133 LN
134♦LBL 05
135 RCL 05
136 *
137 RCL 06
138 +
139 GTO 00
140♦LBL 88
141 "⊢="
142 ARCL X
143 AVIEW
144 RTN
```

Inverse transform

Coefficient of Determination

Calculate r^2.

Input x to calculate y.

```
145♦LBL a
146 GTO IND
08
147♦LBL "INI
T"
148 CLRG
149 CF 00
150 CF 01
151 CF 02
152 SF 21
153 SF 27
154 CF 29
155 RTN
```

Re-initialize.

For initializing.

Important status
Size = 016
Σ = 10
Fix 2

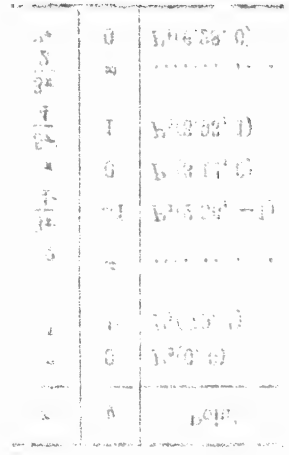
Flags used
F00
F01
F02
F21
F27
F29
F55

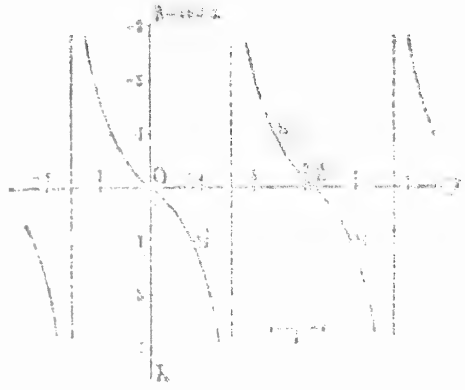
R10 = Σx
R11 = Σx^2
R12 = Σy
R13 = Σy^2
R14 = Σxy
R15 = n

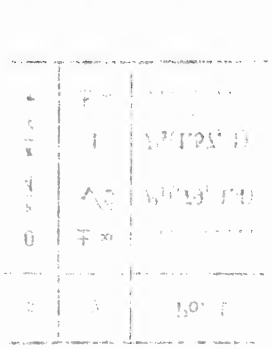
Notes

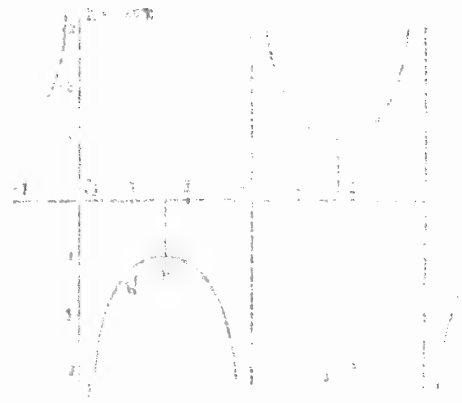
CURVE FITTING

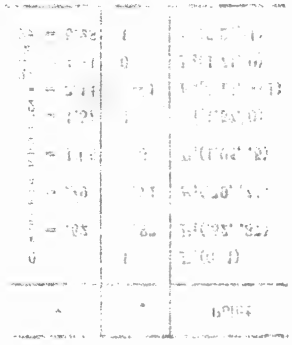
```
01♦LBL "LIN"
02 5
03 "LIN"
04 GTO 13
05♦LBL "EXP"
06 6
07 "EXP"
08 GTO 13
09♦LBL "LOG"
10 7
11 "LOG"
12 GTO 13
13♦LBL "POW"
14 8
15 "POW"
16♦LBL 13
17 FC? 55
18 SF 21
19 STO 00
20 ASTO 03
21 CLΣ
22 ADV
23 AVIEW
24 STOP
25♦LBL C
26 X<>Y
27 XEQ IND 00
28 Σ-
29 STOP
30♦LBL A
31 X<>Y
32 XEQ IND 00
33 Σ+
34 STOP
35♦LBL 07
36 LN
37 RTN
38♦LBL 09
39 LN
40♦LBL 06
41 X<>Y
42 LN
43 X<>Y
44 RTN
45♦LBL E
46 RCL 16
47 RCL 12
48 RCL 11
49 RCL 11
50 XEQ 09
51 STO 03
52 RCL 13
53 RCL 12
54 RCL 11
55 RCL 15
56 XEQ 09
57 RCL 03
58 /
59 STO 04
60 XEQ IND 00
61 STO 06
62 RCL 16
63 RCL 15
64 RCL 11
65 RCL 13
66 XEQ 09
67 RCL 03
68 /
69 STO 05
70♦LBL 03
71 RCL 04
72 RCL 13
73 *
74 RCL 05
75 RCL 15
76 *
77 +
78 RCL 13
79 X↑2
80 RCL 16
81 /
82 STO 09
83 -
84 RCL 14
85 RCL 09
86 -
87 /
88 "R2"
89 XEQ 01
90 RCL 06
91 "a"
92 XEQ 01
93 RCL 05
94 "b"
95 GTO 01
96♦LBL 06
97♦LBL 08
98 E↑X
99♦LBL 05
100♦LBL 07
101 RTN
102♦LBL 09
103 *
104 STO 07
105 RDN
106 *
107 RCL 07
108 -
109 RTN
110 FC? 55
111 CF 21
112♦LBL 00
113 "Y."
114♦LBL 01
115 "⊢="
116 ARCL X
117 AVIEW
118 RTN
119♦LBL 04
120 GTO IND 00
121♦LBL 08
122 RCL 05
123 Y↑X
124 GTO 09
125♦LBL 06
126 RCL 05
127 *
128 E↑X
129♦LBL 09
130 RCL 06
131 *
132 GTO 00
133♦LBL 07
134 LN
135♦LBL 05
136 RCL 05
137 *
138 RCL 06
139 +
140 GTO 00
141♦LBL a
142 GTO IND 08
143 END
```

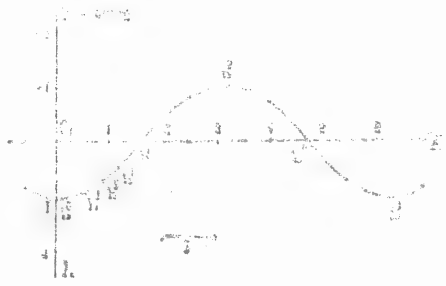


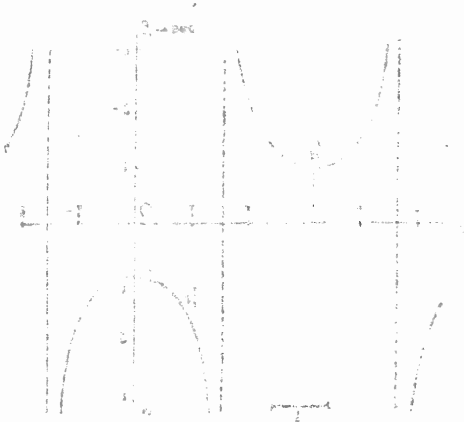


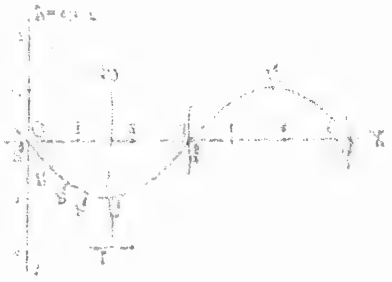


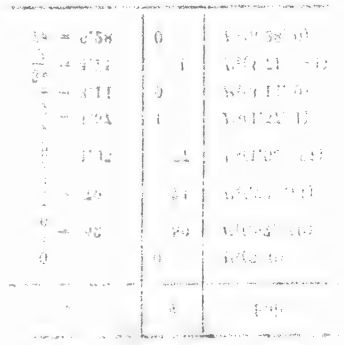


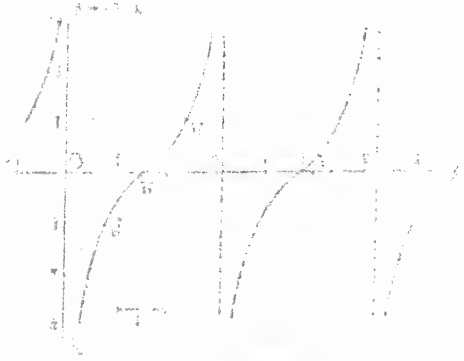


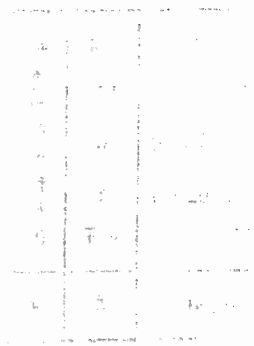




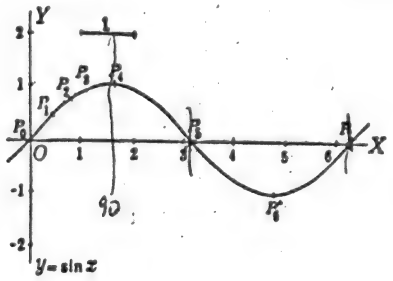




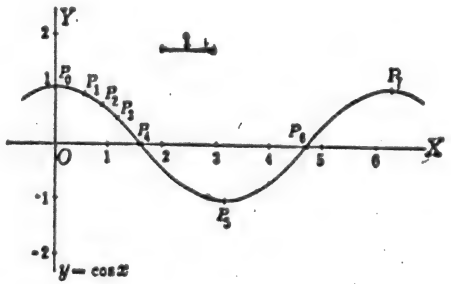




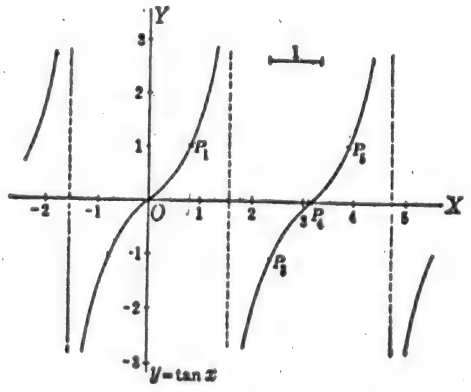
*GRAPHS OF THE TRIGONOMETRIC FUNCTIONS



x	y	Point
0	0	$P_0(0, 0)$
$\frac{\pi}{6} = .52$	.50	$P_1(.52, .50)$
$\frac{\pi}{4} = .79$	.71	$P_2(.79, .71)$*
$\frac{\pi}{3} = 1.05$	.87	$P_3(1.05, .87)$
$\frac{\pi}{2} = 1.57$	1	$P_4(1.57, 1)$
$\pi = 3.14$	0	$P_5(3.14, 0)$
$\frac{3\pi}{2} = 4.71$	−1	$P_6(4.71, -1)$
$2\pi = 6.28$	0	$P_7(6.28, 0)$

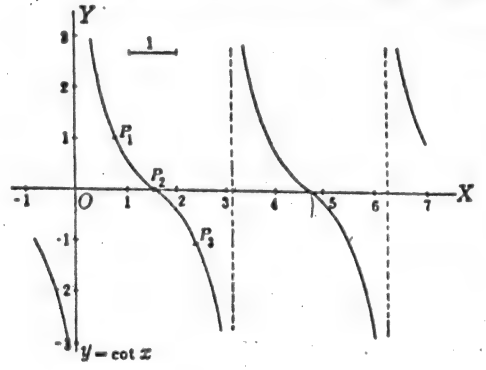


x	y	Point
0	1	$P_0(0, 1)$
$\frac{\pi}{6} = .52$	.87	$P_1(.52, .87)$
$\frac{\pi}{4} = .79$	.71	$P_2(.79, .71)$
$\frac{\pi}{3} = 1.05$	.5	$P_3(1.05, .5)$
$\frac{\pi}{2} = 1.57$	0	$P_4(1.57, 0)$
$\pi = 3.14$	−1	$P_5(3.14, -1)$
$\frac{3\pi}{2} = 4.71$	0	$P_6(4.71, 0)$
$2\pi = 6.28$	1	$P_7(6.28, 1)$

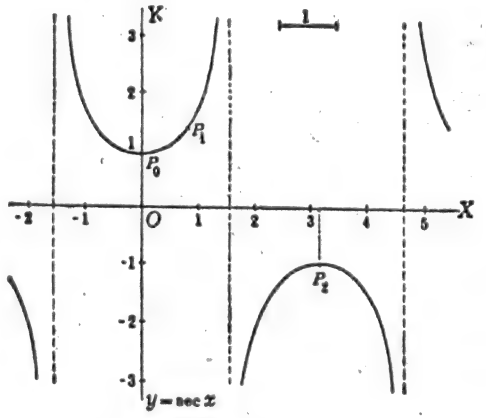


x	y	Point
0	0	$P_0(0, 0)$
$\frac{\pi}{4}$	1	$P_1(.79, 1)$
$\frac{\pi}{2}$	∞	.....
$\frac{3\pi}{4}$	−1	$P_3(2.36, -1)$
π	0	$P_4(3.14, 0)$
$\frac{5\pi}{4}$	1	$P_5(3.93, 1)$
$\frac{3\pi}{2}$	∞	.....
2π	0	$P_8(6.28, 0)$

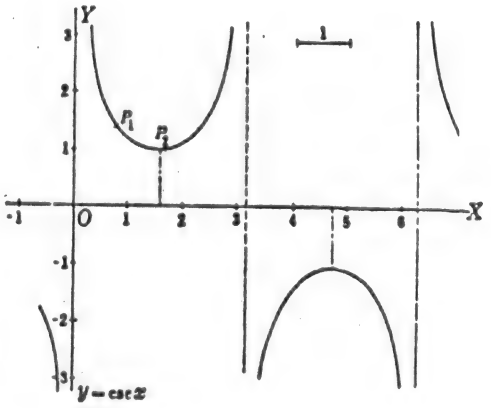
GRAPHS OF THE TRIGONOMETRIC FUNCTIONS



x	y	Point
0	$\pm\infty$	.....
$\frac{\pi}{4}$	1	$P_1(.79, 1)$
$\frac{\pi}{2}$	0	$P_2(1.57, 0)$
$\frac{3\pi}{4}$	−1	$P_3(2.36, -1)$
π	$\pm\infty$	.....



x	y	Point
0	1	$P_0(0, 1)$
$\frac{\pi}{4}$	$\sqrt{2}$	$P_1(.79, 1.4)$
$\frac{\pi}{2}$	$\pm\infty$	.....
π	−1	$P_2(3.14, -1)$



x	y	Point
0	$\pm\infty$	.....
$\frac{\pi}{4}$	$\sqrt{2}$	$P_1(.79, 1.4)$
$\frac{\pi}{2}$	1	$P_2(1.57, 1)$
π	$\pm\infty$	.....

TRIGONOMETRY

TRIGONOMETRIC FUNCTIONS IN A RIGHT-ANGLED TRIANGLE

If A, B, and C are the vertices (C the right angle), and a, b, and h the sides opposite respectively,

$$\text{sine } A = \sin A = \frac{a}{h}, \qquad \text{cosine } A = \cos A = \frac{b}{h},$$

$$\text{tangent } A = \tan A = \frac{a}{b}, \qquad \text{cotangent } A = \cot A = \operatorname{ctn} A = \frac{b}{a},$$

$$\text{secant } A = \sec A = \frac{h}{b}, \qquad \text{cosecant } A = \csc A = \frac{h}{a}.$$

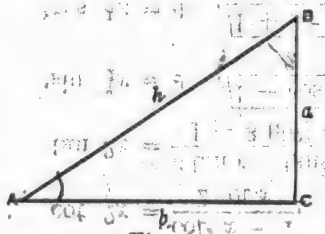


Fig. 4

$$\text{exsecant } A = \text{exsec } A = \sec A - 1$$

$$\text{versine } A = \text{vers } A = 1 - \cos A$$

$$\text{coversine } A = \text{covers } A = 1 - \sin A$$

$$\text{haversine } A = \text{hav } A = \tfrac{1}{2} \text{ vers } A$$

RELATIONS BETWEEN DEGREE OF ACCURACY OF COMPUTED LENGTHS AND ANGLES

When solving a triangle for any of its parts the following should be observed:

Length to:	(requires)	Angle to:
2 significant digits		nearest 30′ = 0.5°
3 significant digits		nearest 05′ = 0.083°
4 significant digits		nearest 01′ = 0.0167°
5 significant digits		nearest 0.1′ = 0.00167°

SIGNS AND LIMITS OF VALUE ASSUMED BY THE FUNCTIONS

Function	Quadrant I		Quadrant II		Quadrant III		Quadrant IV	
	Sign	Value	Sign	Value	Sign	Value	Sign	Value
sin	+	0 to 1	+	1 to 0	−	0 to 1	−	1 to 0
cos	+	1 to 0	−	0 to 1	−	1 to 0	+	0 to 1
tan	+	0 to ∞	−	∞ to 0	+	0 to ∞	−	∞ to 0
cot	+	∞ to 0	−	0 to ∞	+	∞ to 0	−	0 to ∞
sec	+	1 to ∞	−	∞ to 1	−	1 to ∞	+	∞ to 1
cosec	+	∞ to 1	+	1 to ∞	−	∞ to 1	−	1 to ∞

VALUE OF THE FUNCTIONS OF VARIOUS ANGLES

	0°	30°	45°	60°	90°	180°	270°
sin	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1	0	−1
cos	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0	−1	0
tan	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	∞	0	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{3}\sqrt{3}$	0	∞	0
sec	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	∞	−1	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	∞	−1

EXPONENTIAL DEFINITIONS OF CIRCULAR FUNCTIONS

$$\sin x = \frac{1}{2i}(e^{ix} - e^{-ix}) \qquad \text{cosec } x = \frac{2i}{e^{ix} - e^{-ix}}$$

$$(i^2 = -1)$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \sec x = \frac{2}{e^{ix} + e^{-ix}}$$

$$\tan x = \frac{e^{ix} - e^{-ix}}{ie^{ix} + ie^{-ix}} \qquad \cot x = \frac{ie^{ix} + ie^{-ix}}{e^{ix} - e^{-ix}}$$

RELATIONS OF THE FUNCTIONS

$$\sin x = \frac{1}{\text{cosec } x}. \qquad \text{cosec } x = \frac{1}{\sin x}.$$

$$\cos x = \frac{1}{\sec x}. \qquad \sec x = \frac{1}{\cos x}.$$

$$\tan x = \frac{1}{\cot x} = \frac{\sin x}{\cos x}. \qquad \sin^2 x + \cos^2 x = 1.$$

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}. \qquad 1 + \tan^2 x = \sec^2 x.$$

$$1 + \cot^2 x = \text{cosec}^2 x.$$

$$*\sin x = \pm\sqrt{1 - \cos^2 x}. \qquad *\cos x = \pm\sqrt{1 - \sin^2 x}.$$

$$*\tan x = \pm\sqrt{\sec^2 x - 1}. \qquad *\sec x = \pm\sqrt{\tan^2 x + 1}.$$

$$*\cot x = \pm\sqrt{\text{cosec}^2 x - 1}. \qquad *\text{cosec } x = \pm\sqrt{\cot^2 x + 1}.$$

$$\sin x = \cos(90° - x) = \sin(180° - x).$$

$$\cos x = \sin(90° - x) = -\cos(180° - x).$$

$$\tan x = \cot(90° - x) = -\tan(180° - x).$$

$$\cot x = \tan(90° - x) = -\cot(180° - x).$$

$$\text{cosec } x = \cot\frac{x}{2} - \cot x.$$

* The sign in front of radical depends on quadrant in which x falls.

FUNCTIONS OF SUMS OF ANGLES

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y.$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y.$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}.$$

FUNCTIONS OF MULTIPLE ANGLES

$$\sin 2x = 2 \sin x \cos x.$$
$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x.$$
$$\sin 3x = 3 \sin x - 4 \sin^3 x.$$
$$\cos 3x = 4 \cos^3 x - 3 \cos x.$$
$$\sin 4x = 8 \cos^3 x \sin x - 4 \cos x \sin x.$$
$$\cos 4x = 8 \cos^4 x - 8 \cos^2 x + 1.$$
$$\sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x.$$
$$\cos 5x = 16 \cos^5 x - 20 \cos^3 x + 5 \cos x.$$
$$\sin 6x = 32 \cos^5 x \sin x - 32 \cos^3 x \sin x + 6 \cos x \sin x.$$
$$\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1.$$
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}.$$
$$\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}.$$
$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}.$$
$$*\sin \tfrac{1}{2}x = \pm \sqrt{\frac{1 - \cos x}{2}}.$$
$$*\cos \tfrac{1}{2}x = \pm \sqrt{\frac{1 + \cos x}{2}}.$$
$$*\tan \tfrac{1}{2}x = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}.$$

* The sign in front of radical depends on quadrant in which x falls.

MISCELLANEOUS RELATIONS

$$\sin x \pm \sin y = 2 \sin \tfrac{1}{2}(x \pm y) \cdot \cos \tfrac{1}{2}(x \mp y).$$
$$\cos x + \cos y = 2 \cos \tfrac{1}{2}(x + y) \cdot \cos \tfrac{1}{2}(x - y).$$
$$\cos x - \cos y = -2 \sin \tfrac{1}{2}(x + y) \cdot \sin \tfrac{1}{2}(x - y).$$
$$\sin x \cos y = \tfrac{1}{2}[\sin(x + y) + \sin(x - y)]$$
$$\cos x \sin y = \tfrac{1}{2}[\sin(x + y) - \sin(x - y)]$$
$$\cos x \cos y = \tfrac{1}{2}[\cos(x + y) + \cos(x - y)]$$
$$\sin x \sin y = \tfrac{1}{2}[\cos(x - y) - \cos(x + y)]$$
$$\tan x \pm \tan y = \frac{\sin(x \pm y)}{\cos x \cdot \cos y}. \quad \cot x \pm \cot y = \frac{\pm \sin(x \pm y)}{\sin x \cdot \sin y}.$$
$$\frac{1 + \tan x}{1 - \tan x} = \tan(45° + x). \quad \frac{\cot x + 1}{\cot x - 1} = \cot(45° - x).$$
$$\frac{\sin x \pm \sin y}{\cos x + \cos y} = \tan \tfrac{1}{2}(x \pm y).$$
$$\frac{\sin x \pm \sin y}{\cos x - \cos y} = -\cot \tfrac{1}{2}(x \mp y).$$
$$\frac{\sin x + \sin y}{\sin x - \sin y} = \frac{\tan \tfrac{1}{2}(x + y)}{\tan \tfrac{1}{2}(x - y)}.$$
$$\sin^2 x - \sin^2 y = \sin(x + y) \cdot \sin(x - y).$$
$$\cos^2 x - \cos^2 y = -\sin(x + y) \sin(x - y).$$
$$\cos^2 x - \sin^2 y = \cos(x + y) \cos(x - y).$$

INVERSE TRIGONOMETRIC FUNCTIONS

The following table lists each of the six inverse trigonometric functions together with the interval of its principal value:

Function	Interval containing principal value	
	x positive or zero	x negative
$y = \sin^{-1} x$ and $\tan^{-1} x$	$0 \leqq y \leqq \pi/2$	$-\pi/2 \leqq y < 0$
$y = \cos^{-1} x$ and $\cot^{-1} x$	$0 \leqq y \leqq \pi/2$	$\pi/2 < y \leqq \pi$
$y = \sec^{-1} x$ and $\csc^{-1} x$	$0 \leqq y \leqq \pi/2$	$-\pi \leqq y \leqq -\pi/2$

Usually the first letter in "arc" or the name of the inverse trigonometric functions is capitalized if the principal value is desired. Thus

$$\text{Arc sin } \frac{1}{2} = \text{Sin}^{-1} \frac{1}{2} = \frac{\pi}{6},$$

while

$$\text{arc sin } \frac{1}{2} = \frac{\pi}{6} + 2\pi n \text{ or } \frac{5\pi}{6} + 2\pi n$$

In the calculus both for differentiation or integral formulas, capitalization is not adhered to strictly, but principal values are always understood for inverse trigonometric functions when used unless specifically stated otherwise.

RELATIONS BETWEEN SIDES AND ANGLES OF ANY PLANE TRIANGLE

In a triangle with angles A, B, and C and sides opposite a, b, and c respectively,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \text{diameter of the circumscribed circle.}$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$a = b \cos C + c \cos B.$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

$$\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}.$$

$$\sin A = \frac{2}{bc}\sqrt{s(s-a)\,(s-b)\,(s-c)},$$

where $s = \frac{1}{2}(a+b+c)$ and $r = \sqrt{\dfrac{(s-a)\,(s-b)\,(s-c)}{s}}$.

$$\sin \frac{A}{2} = \sqrt{\frac{(s-b)\,(s-c)}{bc}}.$$

$$\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}.$$

$$\tan \frac{A}{2} = \sqrt{\frac{(s-b)\,(s-c)}{s(s-a)}} = \frac{r}{s-a}.$$

$$\frac{a+b}{a-b} = \frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A-B)}.$$

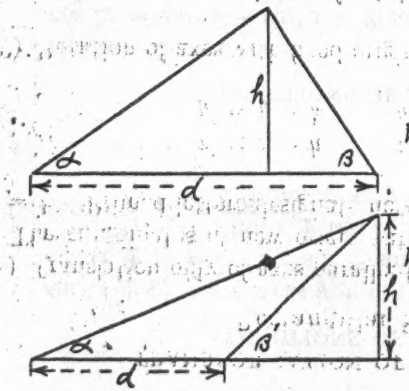


Fig. 5

$$h = d\,\frac{\sin\alpha \sin\beta}{\sin(\alpha+\beta)} = \frac{d}{\cot\alpha + \cot\beta}$$

Similarly

$$h = d\,\frac{\sin\alpha \sin\beta'}{\sin(\beta'-\alpha)} = \frac{d}{\cot\alpha - \cot\beta'}$$

RELATIONS IN ANY SPHERICAL TRIANGLE

If A, B and C be the three angles and a, b, and c the opposite sides,

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A = \frac{\cos b \cos(c \pm \theta)}{\cos \theta}$$

where $\tan \theta = \tan b \cos A$.

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a.$$

$$\sin \tfrac{1}{2} A = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}$$

where $s = \frac{1}{2}(a+b+c)$.

$$\cos \tfrac{1}{2} A = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c}}.$$

$$\tan \tfrac{1}{2} A = \frac{r}{\sin(s-a)}$$

where $r = \sqrt{\dfrac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}$

$$\cos \tfrac{1}{2} a = \sqrt{\frac{\cos(S-B)\cos(S-C)}{\sin B \sin C}}$$

where $S = \frac{1}{2}(A+B+C)$.

$$\sin \tfrac{1}{2} a = \sqrt{\frac{-\cos S \cos(S-A)}{\sin B \sin C}}.$$

$$\tan \tfrac{1}{2} a = R \cos(S-A)$$

where $R = \sqrt{\dfrac{-\cos S}{\cos(S-A)\cos(S-B)\cos(S-C)}}$.

$$\frac{\tan \frac{a+b}{2}}{\tan \frac{c}{2}} = \frac{\cos \frac{A-B}{2}}{\cos \frac{A+B}{2}}, \qquad \frac{\tan \frac{A+B}{2}}{\cot \frac{C}{2}} = \frac{\cos \frac{a-b}{2}}{\cos \frac{a+b}{2}}.$$

$$\frac{\tan \frac{a-b}{2}}{\tan \frac{c}{2}} = \frac{\sin \frac{A-B}{2}}{\sin \frac{A+B}{2}}, \qquad \frac{\tan \frac{A-B}{2}}{\cot \frac{C}{2}} = \frac{\sin \frac{a-b}{2}}{\sin \frac{a+b}{2}}.$$

$$\operatorname{hav} a = \operatorname{hav}(b-c) + \sin b \sin c \operatorname{hav} A$$

$$\operatorname{hav} A = \frac{\sqrt{\operatorname{hav}[a+(b-c)]\operatorname{hav}[a-(b-c)]}}{\sin b \sin c}$$

TRANSFORMATION OF COORDINATES

Rectangular System

(1) Translation only of axes parallel to themselves.
The coordinates of new origin with respect to old axes: $x = h$, $y = k$. Primed letters designate new coordinates.

$$x' = x - h \qquad x = x' + h$$
$$y' = y - k \qquad y = y' + k$$

(2) Rotation of axes with fixed origin.

Angle of rotation $= \theta$

$$x' = x\cos\theta + y\sin\theta \qquad x = x'\cos\theta - y'\sin\theta$$
$$y' = y\cos\theta - x\sin\theta \qquad y = y'\cos\theta + x'\sin\theta$$

(3) Origin translated and axes rotated.
Symbols same as above

$$x' = (x - h)\cos\theta + (y - k)\sin\theta$$
$$y' = (y - k)\cos\theta - (x - h)\sin\theta$$
$$x = x'\cos\theta - y'\sin\theta + h$$
$$y = y'\cos\theta + x'\sin\theta + k$$

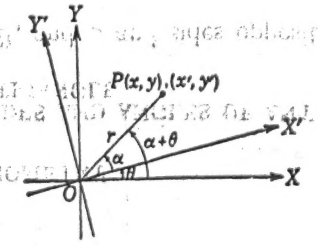
Relation between Rectangular and Polar Coordinates

$x = r\cos\theta \qquad r = \sqrt{x^2 + y^2} \qquad \sin\theta = \dfrac{y}{\sqrt{x^2 + y^2}}$

$y = r\sin\theta \qquad \theta = \tan^{-1}\dfrac{y}{x} \qquad \cos\theta = \dfrac{x}{\sqrt{x^2 + y^2}}$

Rotation of Polar Axes through Angle θ

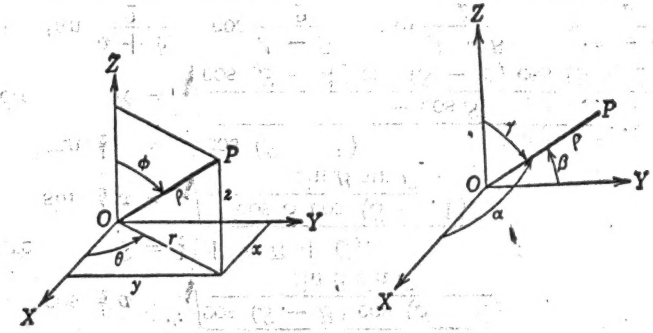
$$x' = r\cos\alpha$$
$$y' = r\sin\alpha$$
$$x = r\cos(\alpha + \theta)$$
$$y = r\sin(\alpha + \theta)$$



TRANSFORMATION OF COORDINATES (Continued)
SOLID ANALYTICAL GEOMETRY

Space Coordinates

1. Rectangular System x, y, z.
2. Cylindrical System r, θ, z.
3. Spherical System ρ, θ, ϕ. In certain situations it is desirable to interchange the symbols ϕ for θ and vice versa.
4. Polar Space System ρ, α, β, γ.



Relations of Coordinates of Systems in Terms of x, y, z

Cylindrical	Spherical	Polar Space
$r = \sqrt{x^2 + y^2}$	$\rho = \sqrt{x^2 + y^2 + z^2}$	$\rho = \sqrt{x^2 + y^2 + z^2}$
$\theta = \tan^{-1}\dfrac{y}{x}$	$\theta = \tan^{-1}\dfrac{y}{x}$	$\alpha = \cos^{-1}\left(\dfrac{x}{\sqrt{x^2 + y^2 + z^2}}\right)$
$z = z$	$\phi = \cos^{-1}\left(\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$	$\beta = \cos^{-1}\left(\dfrac{y}{\sqrt{x^2 + y^2 + z^2}}\right)$
		$\gamma = \cos^{-1}\left(\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$

Relations of Rectangular Coordinates (x, y, z) in Terms of Cylindrical, Spherical and Polar Space Coordinates

Cylindrical	Spherical	Polar Space
$x = r\cos\theta$ $y = r\sin\theta$ $z = z$	$x = \rho\sin\phi\cos\theta$ $y = \rho\sin\phi\sin\theta$ $z = \rho\cos\phi$	$x = \rho\cos\alpha$ $y = \rho\cos\beta$ $z = \rho\cos\gamma$